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EXERCISES ON MECHANICS
AND
NATURAL PHILOSOPHY;
OR
An Easy Introduction to Engineering,

FOR THE USE OF SCHOOLS AND PRIVATE STUDENTS;

CONTAINING

VARIOUS APPLICATIONS OF THE PRINCIPLE OF WORK;

THE THEORY OF THE STEAM ENGINE,
WITH SIMPLE MACHINES;

THEOREMS AND PROBLEMS ON ACCUMULATED WORK;

THE EQUILIBRIUM OF STRUCTURE WITH THE THEORY OF THE ARCH;

THE PRESSURE AND EFFLUX OF FLUIDS;

CALCULATIONS ON RAILWAY CUTTINGS;

&c. &c.

By THOMAS TATE,

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PREFACE.

The present work is intended to supply Elementary Teachers, and Students of Engineering, with simple, and at the same time demonstrative, expositions of the great leading principles of Practical Mechanics and Natural Philosophy. With the view of rendering the subject available for elementary instruction, the demonstrations are, for the most part, conducted on purely arithmetical principles; and numerous problems are given throughout the work as exercises for the pupil.

I have consulted the able writings of Poncelet, Pambour, Morin, and Whewell; but the chief source of my information has been "The Mechanical Principles of Engineering and Architecture," by Professor Moseley, —indeed not a few of the subjects explained in the present volume, were adopted in consequence of the suggestions of this distinguished mathematician. Those who wish to pursue the subject farther, must study the great work just referred to.

Without affecting any great claim to originality, I flatter myself that the scientific reader will here and there find something like new applications of the principles of Work. The formula which I have given for finding the centre of gravity of an arch, cannot fail of meeting with the approbation of practical engineers, on account of its precision, and the labor which it is calculated to save, in determining the equilibrium of the arch; and I may also venture to say, that the simple and arithmetical form given to many difficult problems on WORK, especially those on the Steam Engine, will merit the attention of teachers, and students of engineering.

If this work should prove,—as it is designed to be,—a useful contribution to the educational writings of our times, it will afford me the highest pleasure, as it will give me the proudest recompense.

T. TATE.

Battersea, *May*, 1846.

EXERCISES IN MECHANICS,

&c.

PRELIMINARY IDEAS ON WORK.

1. A horse, or any animal, does work when he moves with a loaded cart, or when he gives motion to any kind of machinery. A steam engine does work when it is used to lift water, or to drive a train of carriages along a railway, or to do in fact any sort of labour for which animals are employed. A man may work with his mind as well as with his body,—when he works with his mind he performs intellectual labour, or the work of a reasoning being,—when he merely works with his body he performs physical labour, or mechanical work which could be done quite as well by a steam engine or any other labouring agent. It is only the latter kind of work that will be considered in this treatise.

Now it will easily be understood, that whenever mechanical work is done, there is a pressure or resistance exerted, and a certain space through which that pressure or resistance is exerted. Thus, when a carpenter saws a piece of timber he applies a pressure to the saw, and this occasions a resistance on the part of the timber to the motion of the saw. Now, whatever may be the pressure applied to the saw, if it were not moved, there would obviously be no work done, and on the other hand, if the saw were moved without any pressure being applied to it, there would also be no work done. When a man carries a weight up a ladder or staircase, we say that he does work; but if he stood still with his load, then, although he would sustain the same pressure, yet he would not do any work. Hence, in order to have work performed, we must have not only pressure, but that pressure must be sustained through a certain space. It is easy to form a rude idea of the comparative quantity of work which two labouring agents might perform:—thus, if two men were to carry the same weight of material to the same height, then they would do the same work; and if the one were to carry one-half the weight to four times the height, then he would do twice the useful work of the other. But it is necessary that we should have some more definite means of estimating work than such methods of calculation afford.

As we measure a distance by the number of times that some unit of length, such as a foot or yard, is contained in the proposed distance; or as we express the weight of any substance by the number of times that some unit of weight, as a pound or a cwt., is contained in the proposed weight; so we must fix upon some unit whereby to express the amount of labour or work which any agent may perform. Now the measuring unit must always be of the same sort or kind as the thing to be measured: thus we measure length by a unit of length, surface by a unit of surface, weight by a unit of weight, and so on. The measure of work, therefore, must be some unit of work.

UNIT OF WORK.

2. The unit of work used in this country, is the labour requisite to raise one pound through the space of one foot. Thus, if a man take a pound weight in his hand and raise it one foot, he will perform one unit of work; if he raise it two feet, he will perform two units of work; and if he raise it three feet, he will perform three units, and so on. Now if he take a four pounds weight in his hand, and raise it five feet, he will perform twenty units of work, because in raising one pound five feet, he will perform five units; therefore in raising four pounds the same height, he will perform four times five units, or twenty units. Here then we observe, that in order to obtain the work expended in raising any body, we multiply the weight of the body in lbs. by the vertical space in feet through which it is raised.

As resistances and pressures of every kind may be expressed in pounds, it follows that the unit of work here laid down will measure every kind of work. It will also afterwards be seen that a unit of work is done whenever one lb. pressure is exerted through a space of one foot, no matter in what direction that space may lie.

Examples.

Ex. 1. Required the units of work expended in raising a weight of 60 lbs. to the height of 23 feet.

Units of work in raising 1 lb. 23 ft. = 23.

∴ ,, 60 lbs. = 60 times 23 = 1380.

Ex. 2. The ram of a pile engine weighs 7 cwt., and has a fall of 23 feet; it is required to find the work performed in raising the ram.

Weight of the ram in lbs. = 7 × 112 = 784 lbs.

Units of work in raising 1 lb. 23 ft. = 23

∴ ,, 784 lbs. ,, = 784 times 23 = 18032

Ex. 3. Required the work as in the last example, when the fall = 19 feet, and the ram = 14 cwts. *Ans.* 29792.

Ex. 4. How many units of work would be required to raise 4 cwts. of coals from a pit whose depth is 70 fathoms?

Weight of coals in lbs. = 4 × 112 = 448 lbs.

Depth of the pit in feet = 70 × 6 = 420 feet.

Work in raising 1 lb. 420 ft. = 420.

∴ ,, 448. lbs. ,, = 448 times 420 = 188160.

Ex 5. Required the work, as in the last example, when the weight = 3 cwts., and depth 25 fathoms. *Ans.* 50400.

Ex. 6. If the weight of a man be 140 lbs., and if he ascend the perpendicular height of 50 feet, how many units of work would he perform?

Here the weight raised is in the man's body = 140 lbs.

∴ Work = 140 × 50 = 7000.

If the man were then to descend by a basket, it is evident, that he would perform the same work upon a counterpoise weight, which he had done in ascending.

Ex. 7. How much work would be required to pump 3000 cubic feet of water from a mine whose depth is 160 fathoms? Here the weight of a cubic foot of water is 62·5 lbs., then,

Weight of water = 62·5 × 3000 = 187500 lbs.

Depth of the mine in ft. = 160 × 6 = 960 feet.

∴ Work = 187500 × 960 = 180000000.

Ex. 8. Required the work, as in the last example, when the cubic feet of water = 250, and the depth = 36 fathoms.

Ans. 3375000

Ex. 9. If a horse draw 104 lbs. out of a well, by means of a cord going over a wheel, moving at the rate of 2½ miles per hour; how many units of work will he perform per minute?

Space moved over per min. $= \frac{2\frac{1}{2} \times 5280}{60} = 220$ ft.

∴ Work per min. = 220 × 104 = 22880.

Ex. 10. Required the work, as in the last example, when the weight = 130 lbs., and the rate = 3 miles per hour.

Ans. 34320.

3. When considerable distances are measured, we use a higher unit than that of feet or yards; so, in like manner, when the work is very great, it is found more convenient to adopt a larger unit of work. Now Watt estimated that a horse could perform 33000 units of work per minute; this work therefore is called a horse power. It will now be easy to express work done, *in a given time*, in units of a horse power, and conversely.

Ex. 1. How many horse powers would it take to raise 1 cwt. of coals per minute from a pit whose depth is 200 fathoms?

Weight of coals in lbs. = 112 lbs.

Depth of the pit in ft. = 200 × 6 = 1200 ft.

As 112 lbs. of coals are raised every minute,

∴ Work done per min. = 112 × 1200 = 134400.

Now a horse does 33000 units of work in this time, therefore as many times as we can take 33000 out of 134400, so many horse powers must we have, that is,

Horse powers, or H.P. $= \frac{134400}{33000} = 4·07$.

Ex. 2. Required the H. P., as in the last example, when the weight = 5 cwts., and depth = 160 fathoms. *Ans.* 16·2.

Ex. 3. How many H. P. would be required to raise 2000 cubic feet of water per hour, from a mine whose depth is 180 fathoms?

Weight of water in lbs. = 62·5 × 2000.

Depth of the mine in ft. = 180 × 6.

$$\therefore \text{Work per min.} = \frac{62{\cdot}5 \times 2000 \times 180 \times 6}{60}$$

$$\text{And H. P.} = \frac{62{\cdot}5 \times 2000 \times 180 \times 6}{60 \times 33000} = 68{\cdot}1.$$

Ex. 4. Required the H. P., as in the last example, when the number feet = 2500, and depth = 86 fathoms.

Ans. 40·7.

Ex. 5. A Winding engine is moved by two horses, what weight of coals will be raised per hour from a pit whose depth is 50 fathoms?

Work done by the horses per hour = 33000 × 2 × 60 = 3960000.

Depth of the pit = 300 feet.

Work in raising 1 lb. of coals = 300.

Now as 300 units of work go to raise 1 lb. of coals, there fore as many times as 300 can be taken out 3960000, so many lbs. must be raised in the given time.

$\therefore$ Number lbs. raised per hour $= \frac{3960000}{300} = 13200$ lbs. or 5·9 tons.

Ex. 6. Required the same, as in the last example, when the engine is moved by 3 horses, and the depth is 40 fathoms.

Ans. 11·04 tons.

Ex. 7. In what time will an engine, capable of performing the work of 8 horses, raise a ton of material from the depth of 170 fathoms?

Work of the engine per min. = 33000 × 8 = 264000.

Work in raising 1 ton = 2240 × 170 × 6 = 2284800.

Here, as the engine does 264000 units of work every minute, therefore as many times as we can take this number out of 2284800, so many minutes will be taken in raising one ton, that is,

Number min. to raise 1 ton $= \frac{2284800}{264000} = 8 \cdot 6$ min.

Ex. 8. Required the same, as in the last example, when the H. P. = 7, and depth = 47 fathoms. *Ans.* 2·7 min.

Ex. 9. How many cubic feet of water will an engine of 20 horse powers raise per hour from a mine whose depth is 80 fathoms?

Work done per hour $= 33000 \times 20 \times 60$.

Work in raising 1 foot of water $= 62 \cdot 5 \times 80 \times 6$.

$\therefore$ Number ft. raised per hour $= \frac{33000 \times 20 \times 60}{62 \cdot 5 \times 80 \times 6} = 1320$.

Ex. 10. When the H. P. = 34, and depth mine = 72 fathoms, required the same, as in the last example.

Ans. 2493.

Ex. 11 From what depth will an engine of 5 horse powers raise 4 tons of coals per hour?

Work done per hour $= 33000 \times 5 \times 60$.

Work in raising 4 tons of coals 1 ft. $= 2240 \times 4 \times 1$.

$\therefore$ Number feet in depth $= \frac{33000 \times 5 \times 60}{2240 \times 4} = 1104$ ft.

Ex. 12. An engine is observed to raise 3 tons of material per hour from a mine whose depth is 120 fathoms; it is required to find the horse powers of the engine, supposing $\frac{1}{5}$ of its work to be lost in transmission.

Work done per min. $= \frac{2240 \times 3 \times 120 \times 6}{60} = 80640$.

Now only $\frac{4}{5}$ of the work of the engine go to raise the material.

$\therefore$ Useful work of one H. P. per min. $= \frac{4}{5} \times 33000$.

And H. P. $= \frac{80640}{\frac{4}{5} \times 33000} = 3 \cdot 05$.

Ex. 13. Required the same, as in the last example, when the weight = 5 tons, and the depth = 210 fathoms.

Ans. 8·9.

Ex. 14. What must be the horse powers of an engine working for 10 hours per day, to supply 3000 families with

80 gallons of water each per day, supposing the water to be raised to the mean height of 70 feet?

Here the water pumped per min. $= \frac{3000 \times 80}{10 \times 60} = 400$ gals.

$\therefore$ Weight „ „ $400 \times 10 = 4000$ lbs.

H. P. $= \frac{4000 \times 70}{33000} = 8 \cdot 4$.

Ex. 15. Required the same as in the last examples, when the time $= 8$ hours, number families $= 5000$, number gallons $= 40$, and the mean height $= 60$ feet. *Ans.* 7·5.

Ex. 16. Required the horse powers of an engine which pumps water from three different levels, whose depths are 40, 70, and 90 fathoms respectively; from the first 20 cubic feet of water are raised per minute, from the second 10 feet, and from the third 35 feet.

Work per min. in the 1st. level $= 62 \cdot 5 \times 20 \times 40 \times 6$.

„ „ 2nd. „ $= 62 \cdot 5 \times 10 \times 70 \times 6$-

„ „ 3rd. „ $= 62 \cdot 5 \times 35 \times 90 \times 6$.

$\therefore$ Total work per min. $= (20 \times 40 + 10 \times 70 + 35 \times 90)$ $62 \cdot 5 \times 6$.

$\therefore$ H. P. $= \frac{4650 \times 62 \cdot 5 \times 6}{33000} = 52 \cdot 8$.

Ex. 17. Required the same as in the last example, when the depths are 20, 50, and 70 fathoms, and the water raised from each, 30, 20, and 10 cubic feet. *Ans.* 26·1

Ex. 18. Required the horse powers of the engine working the pumps, in the last example, supposing $\frac{1}{3}$ of the work to be lost in transmission. *Ans.* 39·2

Ex. 19. How many cubic feet of water will an engine of 12 horse powers pump, per hour, from a pit whose depth is 40 fathoms, allowing that $\frac{1}{3}$ of the work of the engine is destroyed by useless resistances? *Ans.* 1056.

Ex. 20. There were 4000 cubic feet of water in a mine whose depth is 60 fathoms, when an engine of 70 horse powers began to work the pump; now the engine continued to work for 5 hours before the mine was cleared of the water; required the number of feet of water which had run into the mine per hour, supposing $\frac{1}{3}$ of the work of the engine to be lost in transmission.

Effective work of the engine in 5 h. $= 33000 \times 70 \times 60 \times 5 \times \frac{2}{3}$.

Work in raising 1 ft. of water $= 62 \cdot 5 \times 60 \times 6$.

Number feet of water pumped $= \frac{33000 \times 70 \times 60 \times 5 \times 2}{62 \cdot 5 \times 60 \times 6 \times 3} =$ 20533.

$\therefore$ Water run in during 5 h. $= 20533 - 4000 = 16533$ ft.

" " 1 h. $= \frac{16533}{5} = 3306$ ft.

Ex. 21. A forge hammer weighing 2 cwts. makes 60 lifts per minute, now the perpendicular height of each lift is $2\frac{1}{2}$ feet; it is required to find the horse powers of the engine giving motion to the hammer.

Weight of hammer in lbs. $= 112 \times 2 = 224$ lbs.

Work in each lift $= 224 \times 2\frac{1}{2} = 560$.

Work in 60 lifts, or per min. $= 560 \times 60 = 33600$.

$\therefore$ H. P. $= \frac{33600}{33000} = 1 \cdot 02$ nearly.

Ex. 22. Required the same as in the last example, when the weight of the hammer = 10 cwts., number of lifts per min. = 70, and height of each lift = 3 ft. *Ans.* 7·1

Ex. 23. What must be the weight of the hammer in the last example, when the horse powers of the engine = 4?

Ans. 5·6 cwts.

Ex. 24. An engine of 5 horse powers raises 30 cwts. of coals per hour from a pit whose depth is 240 fathoms, and at the same time gives motion to a forge hammer which makes 25 lifts per minute, each lift being 3 feet; it is required to determine the weight of the hammer.

Work done by the engine per min. $= 33000 \times 5 = 165000$.

Work in raising coals per min. $= \frac{30 \times 112 \times 240 \times 6}{60} = 80640$.

$\therefore$ Work in raising hammer per min. $= 165000 - 80640 = 84360$.

Work per min. in raising 1 cwt. of the hammer $= 112 \times 3 \times 25 = 8400$.

$\therefore$ Number of cwts. in the hammer $= \frac{84360}{8400} = 10$ cwts.

WORK OF LIVING AGENTS.

4. The labouring force assigned, by Watt, to a horse has been found by more recent experiments to be considerably too much; the number 22000 appears to be about the work a horse of average strength will perform per minute; however, the number 33000 is still retained by engineers as the unit of a horse power. It is also important to observe, that the labouring force of animals varies very much with the manner in which their muscular strength is exerted; and also, with the rate at which they labour.

5. The following table (chiefly taken from Morin's Mechanique Practique,) shows the greatest amount of effective work which a labourer will perform under the different modes in which he exerts his muscular power.

Work done by a Man per Minute.

Duration of labour—8 hours per day.	
Raising his own body	4250
Working at the Treadmill	3900
Drawing, or pushing horizontally	3120
Pushing and drawing alternately in a vertical direction	2380
Turning a handle	2600
Working with his arms and legs as in rowing	4000
Duration of labour 6 hours per day.	
Raising material with a pulley	1560
Raising material with the hands	1470
Raising material upon the back and returning empty	1126
Duration of labour 10 hours per day.	
Raising materials with a wheel-barrow on ramps	720
Throwing earth to the height of 5 feet	470

6. *To compare the traction of animals with the rate at which they travel.*

The traction, or force with which animals pull, decreases with the increase of speed. The following table, given by Tredgold, shows the relation of speed and traction of a horse.

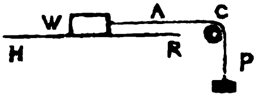
Rate in miles per hour.	Traction in lbs.
2	166
3	125
3½	104
4	83
4½	62½
5	41⅔

The relation exhibited in this table may be generally expressed by the formula, $t = 250 - 41\frac{2}{3}\, r$, where t is the traction in lbs., and r the rate in miles per hour. From this formula it appears, that a horse will do the greatest amount of work when he travels at the rate of three miles per hour.

WORK IN OVERCOMING THE RESISTANCE OF FRICTION.

7. When a body is moved slowly along a horizontal plane, the resistance to be overcome is due to friction. It has been found by experiment that this resistance, on a given surface, is a certain proportional part of the weight of the body; and that it is not affected by any change in the rate of the body's motion, or the extent of the rubbing surfaces.

When a cart is drawn along a *very good* McAdamized road, the resistance of friction is about $\frac{1}{30}$ of the whole load, or about 74 lbs. per ton; so that a horse, in order to draw a ton along such a road, must pull with the force of 74 lbs. This is called the traction of the horse. A carriage upon a railway only requires a pressure of $\frac{1}{280}$ part of the weight to give it motion, or about 8 lbs. per ton. The fractions $\frac{1}{30}$ and $\frac{1}{280}$ are called the coefficients of frictions. As the rubbing surfaces become smoother, these coefficients become smaller.



Let W be a weight drawn upon the horizontal plane H R, by means of a weight P attached to a cord going over the wheel C; then the weight, P, just necessary to draw W along the plane will be equal to the resistance of friction. If W be 1 ton, then, in the case of the railway, P will be 8 lbs.. and if W be 2 tons, P will be 2 times 8 lbs. or 16 lbs., and so on. Now, whatever distance P descends, the weight W will be drawn along the plane the same distance; hence the work done upon W will be the weight of P in lbs. multiplied by the distance in feet through which it descends,—or, what is the same thing,—the resistance of friction in lbs. multiplied by the space in feet over which W is moved upon the plane.

8. The work applied to a machine is consumed by the work done, or the useful work, together with the useless work, or the work destroyed by the friction of the parts of the machine. When the work applied exceeds the work consumed, the redundant work goes to increase the speed of the parts of the machine, which thus act as a reservoir of work. This motion will go on increasing until the work of the resistance and the useful work are together equal to the work applied, and then the motion of the machine will become uniform. Thus, in a railway carriage, at first the work of the engine exceeds the work of the resistances, and therefore the speed of the engine goes on increasing; but, as the speed increases, the work of the resistances also increases, so that at length the engine attains a nearly uniform motion, which is called the greatest or maximum speed, and then the work destroyed by the resistances will be exactly equal to the work applied by the moving power. It is on this principle that the following calculations are made.

Work in moving a carriage upon a horizontal plane.

Ex. 1. What must be the effective H. P. of a locomotive engine, which moves at the steady speed of 30 miles per hour, upon a level rail, the weight of the train being 50 tons, and the friction 8 lbs. per ton, the resistance of the atmosphere being neglected?

Resistance of friction $= 8 \times 50 = 400$ lbs.

Distance moved in ft. per min. $= \frac{30 \times 5280}{60} = 2640$.

Work of friction per min. $= 400 \times 2640$.

Now as the speed of the train is uniform, the work of the resistances will be equal to the effective work of the engine,

$\therefore$ H. P. engine $= \frac{400 \times 2640}{33000} = 32$.

Ex. 2. Required the same as in the last example when the speed = 50 miles, and the weight = 60 tons. *Ans.* 64.

Ex. 3. At what rate, in miles per hour, will a train of 80 tons be drawn by an engine of 70 H. P.?

Work done by the engine per hour $= 70 \times 33000 \times 60$.

Work consumed in moving the train 1 mile $= 8 \times 80 \times 5280$.

$\therefore$ Number miles per hour $= \frac{70 \times 33000 \times 60}{8 \times 80 \times 5280} = 41{\cdot}01$ miles.

Ex. 4. Required the same as in the last example, when the weight = 90 tons, and the H. P. = 60. *Ans.* 31·2.

Ex. 5. An engine of 40 H. P. moves with the maximum speed of 35 miles upon a level rail; required the gross load of the train.

Work engine per hour $= 40 \times 33000 \times 60$.

Work consumed per hour in moving 1 ton $= 8 \times 35 \times 5280$.

$\therefore$ Number tons $= \frac{40 \times 33000 \times 60}{8 \times 35 \times 5280} = 53{\cdot}5$.

Ex. 6. Required the same as in the last example, when the H. P. = 80, and the speed = 45 miles. *Ans.* 83·3.

Ex. 7. In what time will an engine of 50 H. P., moving a train of 90 tons, complete a journey of 40 miles?

Work expended in moving the train 40 miles $= 8 \times 90 \times 40 \times 5280$.

Work engine per hour $= 50 \times 33000 \times 60$.

$\therefore$ Number hours $= \frac{8 \times 90 \times 40 \times 5280}{50 \times 33000 \times 60} = 1{\cdot}53$.

Ex. 8. Required the same as in the last example, when the H. P. = 60, weight train = 70 tons, and distance = 330 miles. *Ans.* 8·2.

Ex. 9. What work per minute will a horse perform when travelling at the rate of 2 miles per hour?

Here, by the table Art. 6, the traction = 166 lbs.

$\therefore$ Work per min. $= \frac{2 \times 5280 \times 166}{60} = 29216$.

Ex. 10. What gross load will a horse draw, travelling at 3 miles per hour, upon a road whose friction is $\frac{1}{12}$ of the whole load?

Here the traction is 125 lbs., but the gross load must be 12 times this weight, or 1500 lbs. = 13·3 cwts.

Ex. 11. At what rate will a horse draw a ton, on a road whose coefficient of friction is $\frac{1}{30}$?

Here, traction $= \frac{2240}{30} = 74$ lbs.

Therefore, by the table, the rate must be between 4 and 4½ miles.

Ex. 12. If a horse exert a traction of 160 lbs. when moving at the rate of 2½ miles per hour, what gross weight will he draw upon a road whose coefficient of friction is $\frac{1}{12}$, and what work will he perform per min.?

Here the traction is $\frac{1}{12}$ of the load,

$\therefore$ Load = 12 times 160 lbs. = 17·1 cwts.

Distance moved in ft. per min. $= \frac{2\frac{1}{2} \times 5280}{60} = 220$.

$\therefore$ Work done per min. $= 160 \times 220 = 35200$.

Ex. 13. If a horse draw a gross load of 1½ tons upon a road whose coefficient of friction is $\frac{1}{30}$, what traction will he exert?

Weight moved in lbs. $= 1\frac{1}{2} \times 2240 = 3360$ lbs.

Now the resistance of friction, or the resistance which the horse has to overcome is $\frac{1}{20}$ of this weight,

$\therefore$ Traction $= \frac{1}{20}$ of 3360 lbs. $= 168$ lbs.

Ex. 14. Required the same as in the last example, when the gross load $= 1\frac{3}{4}$ tons, and the coefficient of friction $= \frac{1}{30}$?

Ans. 130⅔ *lbs.*

Ex 15. In how many days of 8 hours long will a horse whose traction is 160 lbs., transport 12000 cubic feet of material, weighing 200 lbs. each cubic foot, to the mean distance of 4 miles, on a road whose coefficient of friction is $\frac{1}{30}$, supposing the weight of the cart to be $\frac{1}{5}$ of the gross load, and that the horse travels with the full load at the rate of $2\frac{1}{2}$ miles per hour, and returns with the empty cart at the rate of 5 miles per hour; required also the cost of the transport, allowing 10*s.* per day for the driver and horse.

Gross load $= 160 \times 30 = 4800$ lbs.

Net load $= 4800 \times \frac{4}{5} = 3840$ lbs.

Weight of the material $= 12000 \times 200 = 2400000$ lbs.

$\therefore$ Number of loads $= \frac{2400000}{3840} = 625$.

Time in going and returning with 1 load $= \frac{4}{2\frac{1}{2}} + \frac{4}{5} = 2{\cdot}4$ hours.

$\therefore$ Time in removing the whole material $= 2{\cdot}4 \times 625 = 1500$ hours.

$\therefore$ Number of day's work $= \frac{1500}{8} = 187{\cdot}5$.

Cost $= 187{\cdot}5 \times 10s. = £93.\ 15s.$

Ex. 16. How many horses would be required to complete the transport, of the last example, in 10 days?

From the last example, we find that it would take 187·5 horses to complete the work in 1 day,

$\therefore$ Number of horses to do the work in 10 days $= \frac{1}{10}$ of $187{\cdot}5 = 18{\cdot}75$.

Ex. 17. Required the same as in Ex. 15, when the material $= 24000$ cubic feet, the weight of a cubic foot $= 120$ lbs., and the mean distance = 3 miles, the other data remaining the same. *Ans.* 168·75 *days*, and cost £84. 7*s.* 6*d.*

Work of the Saw Machine.

9. The resistance to be overcome in the process of sawing is occasioned by the friction of the teeth of the saw upon the timber. According to Navier about 29000 units of work are required to saw a square foot of green oak planking. This datum will enable us to calculate the various elements that may be required in a problem. In the following calculations no time is allowed for the adjusting of the planks.

Ex. 1. What must be the horse powers of an engine to cut 6000 sq. ft. of oak planking in a day of 10 hours long?

Work in cutting 1 foot = 29000.

" " 6000 feet = 29000 × 6000.

$$\therefore \text{Work per min.} = \frac{29000 \times 6000}{10 \times 60} = 290000.$$

$$\text{And H. P.} = \frac{290000}{33000} = 8{\cdot}7 \text{ nearly.}$$

Ex. 2. Required the same as in the last example, when the number sq. ft. = 5600, and the hours per day = 12.

Ans. 6·8.

Ex. 3. In what time will an engine of 2 horse powers cut a plank 9 feet by 1·3 feet?

Work engine per min. = 33000 × 2 = 66000.

Surface of the plank = 1·3 × 9 = 11·7 sq. ft.

Work in cutting the plank = 29000 × 11·7 = 339300.

$$\therefore \text{Number min. in cutting} = \frac{339300}{66000} = 5{\cdot}1.$$

Ex. 4. In what time will an engine of 5 H. P. cut 300 planks, each plank being 20 ft. by 18 in.? *Ans.* 26·3 *hours.*

Ex. 5. How many feet of plank will an engine of 4 horse powers cut in a day of 12 hours long?

Work engine per day = 95040000,

But work in cutting 1 foot = 29000.

$$\therefore \text{No. of feet cut per day} = \frac{95040000}{29000} = 3277{\cdot}2.$$

Ex. 6. If an engine of 4 effective horse powers cut 24 square feet of memel plank in 5 minutes, how many units of work are consumed in cutting 1 square foot.

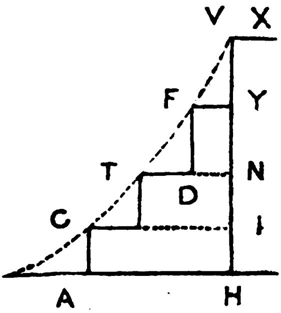
Work engine in 5 min. $= 33000 \times 4 \times 5 = 660000$.

But this result is the work destroyed in cutting 24 square ft.

$\therefore$ Work in cutting 1 foot $= \frac{660000}{24} = 27500$.

WORK IN MOVING A BODY ON AN INCLINE PLANE.

10. If a body be moved along any surface, without friction, the work performed is equal to the product of the weight of the body in lbs. by the vertical height in feet through which it is raised.



Let a body be moved slowly along the line A C L T D F Y V, then the work done will be that which is due to the perpendicular height H V through which the body is raised: because the work done through the vertical line A C is the same as that which would be done through H I; now because C I is a horizontal line, and since there is no friction, it will not require any work to move the body from C to L; in like manner the work done through the vertical line L T will be that which is due to the perpendicular height I N, and the work done on the horizontal line T D will be nothing; and so on; so that the whole work upon the body is simply that which would be performed in raising it through the vertical line H V.

Now since the parts A C, C L, T D, D F, &c., may be taken any relative dimensions, or they may be taken as small as we please, therefore the line A C L T D F Y V may be made to coincide with any assignable figure, or at least to approximate to the form of any curve nearer than any assignable difference.

11. *Work in overcoming the resistances of friction and gravity.*

If the inclination of the plane, upon which a body is moved, is small, the pressure upon the plane will obviously be very nearly equal to the weight of the body; hence the work of friction will be calculated after the manner explained in Art. 7, and that of gravity by Art. 10.

Ex 1. A train of 80 tons ascends an incline which has a rise of 2 feet in every 100 feet, with the uniform speed of 15 miles per hour, what must be the effective horse powers of the engine, the friction being as usual?

Speed train per min. = 1320 ft.

Weight train in lbs. = 179200.

Rise of the rail for 1 ft. = $\frac{2}{100}$ ft.

∴ ,, 1320 ft. = $\frac{2}{100} \times 1320 = 26{\cdot}4$ ft.

Hence it appears that the whole weight of the train is raised 26·4 ft. every minute in opposition to gravity.

∴ Work due to gravity per min. = $179200 \times 26{\cdot}4 = 4730880$.

But work due to friction per min. = $8 \times 80 \times 1320 = 844800$.

∴ Total work per min. = $4730880 + 844800 = 5575680$.

And H. P. = $\frac{5575680}{33000} = 168{\cdot}9$.

Ex. 2. Required the H. P., as in the last example, when the weight of the train = 90 tons, the rise = ½ in 100, and the speed = 20 miles. *Ans.* 92.

Ex. 3. A train of 100 tons ascends an incline which has a rise of $\frac{1}{6}$ in 100; required the maximum speed when the effective horse powers of the engine = 80.

Rise rail in 1 foot =

∴ ,, 5280 feet, or 1 mile = 8·8 ft.

Weight train in lbs. =

∴ Work due to gravity in 1 mile =

,, friction ,, =

∴ Total work done in 1 mile =

Work of the engine per hour =

∴ No. miles per hour $= \frac{\text{Work engine}}{\text{Work in 1 mile}} = 25 \cdot 5$.

Ex. 4. A train of 60 tons ascends an incline which has a rise of $\frac{1}{4}$ in 100, required the maximum speed per hour, when the effective horse powers of the engine = 40. *Ans.* 18·4 *miles.*

Ex. 5. An engine of 50 H. P. ascends a gradient having a rise of $\frac{3}{4}$ in 100, with the steady speed of 20 miles per hour; required the weight of the train in tons. *Ans.* 37·8 *tons.*

This example may be wrought after the method of Ex. 5, Art. 8.

Ex. 6. A train of 100 tons descends a gradient having a rise of $\frac{1}{4}$ in 100, with the steady speed of 60 miles per hour; it is required to determine the H. P. of the engine.

Here, work due to friction per min. = 4224000.

„ work due to gravity „ = 224000 × 13·2 = 2956800.

In this example gravity acts with the engine.

∴ Work engine per min. = 4224000 — 2956800 = 1267200.

And H. P. = 38·4.

Ex. 7. A train of 80 tons descends a gradient having a rise of $\frac{1}{3}$ in 100, with the uniform speed of 50 miles per hour; it is required to determine the H. P. of the engine. *Ans.* 5·6.

Here it will be observed, that the work of gravity acts with the work of the engine.

Ex. 8. If a horse exert a traction of 160 lbs., what weight will he pull up a hill which has a rise of 3 feet in 100 feet, supposing the coefficient of friction to be $\frac{1}{12}$?

Work of the horse in moving over 100 feet = 160 × 100.

Work of friction in moving 1 lb. over 100 ft. $= \frac{1}{12} \times 100$.

„ gravity „ „ $= 1 \times 3$.

Total work in moving 1 lb. over 100 ft. $= \frac{100}{12} + 3 = 11 \cdot 3$.

∴ No. lbs. drawn $= \frac{160 \times 100}{11 \cdot 3}$ lbs. = 12·6 cwts.

Ex. 9. What would be the backward pressure of a horse in going down a hill which has a rise of 5 in 100, with a load of 1 ton, supposing the coefficient of friction to be $\frac{1}{30}$?

Ans. 37·4 *lbs.*

Ex. 10. How many horses would it take to draw a load of 6 tons up a hill having a rise of $2\frac{1}{2}$ in 100, supposing the resistance of friction to be $\frac{1}{12}$ of the whole load, and the traction of each horse 160 lbs.?

Work due to friction in moving 6 tons over 100 ft. =

,, gravity ,, ,, =

Total work in moving 6 tons over 100 ft. =

Work of 1 horse in 100 ft. =

$\therefore$ No. horses $= \frac{\text{Total work}}{\text{Work of 1 horse}} = 9·1$

Ex. 11. What work will a horse perform in transporting a load of 1 ton, on the road in the last example, to the distance of 10 miles, when the total rise is 50 feet?

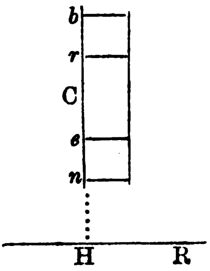
Work due to friction $= \frac{2240}{12} \times 10 \times 5280.$

,, gravity $= 2240 \times 50.$

$\therefore$ Total work $= 9968000.$

WORK IN RAISING MATERIAL HAVING A GIVEN FORM.

12. Let $n\,b$ be the mass raised from the horizontal line H R, C the middle or centre of gravity of the mass, e and r points equidistant from C. Now if we suppose equal weights (say for example 3 lbs.) to be placed at e and r, the centre of gravity of these weights will be at C, then



Work in raising 3 lbs. to $r = 3 \times \mathrm{H}r$.

,, ,, to $e = 3 \times \mathrm{H}e$.

$\therefore$ Total work in raising the material at r and $e = 3 \times \mathrm{H}r + 3 \times \mathrm{H}e = 3(\mathrm{H}r + \mathrm{H}e) = 3 \times 2 \times \mathrm{HC}$.

Now the same may be proved for any two points equi-distant from C, hence *the work in raising the whole material will be its weight, multiplied by the height to which the centre of gravity is raised.*

The demonstration here given may be easily extended to more general cases.

Ex. 1. Required the work in raising the material in a wall 40 ft. long, 16 ft. high, and 2 ft. thick; supposing the weight of a cubic foot of the material to be 150 lbs.

Contents of the wall $= 40 \times 16 \times 2$.

Weight ,, $= 40 \times 16 \times 2 \times 150$.

Now the height of the centre of gravity of the wall will be 8 ft.

$\therefore$ Work $= 40 \times 16 \times 2 \times 150 \times 8 = 1536000$.

Ex. 2. The shaft of a pit is to be sunk 120 feet in depth, and 6 feet in diameter; required the work in raising the material, supposing a cubic foot to weigh 100 lbs.

Number cubic feet in the shaft $= 6^2 \times \cdot 7854 \times 120$.

Weight of the material ,, $= 6^2 \times \cdot 7854 \times 120 \times 100$.
$= 339292 \cdot 8$ lbs.

$$\therefore \text{Work} = 339292 \cdot 8 \times \frac{120}{2} = 20,357,568.$$

Ex. 3. In how many days would a man working with a wheel and axle, raise the material in the last example? *Ans.* 16·3.

Ex. 4. Required the work in raising 3 cwts. of coals from a pit whose depth is 20 fathoms, the circumference of the rope being 2 inches, allowing that the weight of 1 foot of rope of 1 inch in circumference is ·046 lbs.

Weight of 1 ft. of the rope $= 2^2 \times \cdot 046 = \cdot 184$ lbs.

Weight of the whole rope $= \cdot 184 \times 120 = 22 \cdot 08$ lbs.

$$\therefore \text{Work in raising the rope} = 22 \cdot 08 \times \frac{120}{2} = 1324 \cdot 8.$$

Work in raising the coals $= 112 \times 3 \times 120 = 40320$.

$\therefore$ Total work $= 1324 \cdot 8 + 40320 = 41644 \cdot 8$.

Ex. 5. The material in the largest Pyramid, Cheops, is 6 millions of tons, and the perpendicular height is 480 feet; required the bushels of coals which would be consumed in raising the material, supposing that a bushel of fuel, employed in our best steam engines, can perform 70 millions of work; required also the time which an engine of 300 horse powers would take to raise the material.

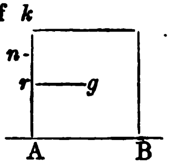
Weight of the material $= 6000000 \times 2240$ lbs.

$\therefore$ Work $= 6000000 \times 2240 \times \frac{480}{4}$

$\therefore$ Number bushels $= \frac{6000000 \times 2240 \times 120}{70000000} = 23040.$

And Time $= \frac{\text{Work}}{33000 \times 300 \times 60 \times 24} = 113$ days.

Ex. 6. The side A B, of a cube of granite is 4 feet, and the weight of a cubic foot is 170 lbs.; it is required to find the work necessary to overturn it on its edges A.



Here the distance, A g, of the centre of gravity from the edge $= \frac{1}{2}\sqrt{2 \times 4^2}$ $= 2{\cdot}828.$

When the cube is about to fall the centre of gravity, g, will be at n, in the vertical line A K.

$\therefore$ the distance, $r\,n$, the centre of gravity will be elevated $= 2.828 - 2 = {\cdot}828$ ft.

$\therefore$ Work $= 64 \times 170 \times {\cdot}828 = 9008.$

Ex. 7. The perpendicular height, D V, of a right pyramid is 12 feet, the side of the base A B, which is square, is 8 feet, and a cubic foot of the material weighs 100 lbs.; it is required to find the work necessary to overturn the pyramid upon its edge A.

When the body is about to fall, the centre of gravity, g, will be at n in the the vertical line A K, and the work in bringing the body to this position, will be due to the vertical distance, $r\,n$, through which the centre of gravity has been raised.

Then D $g =$ A $r = \frac{12}{4} = 3$; A D $= \frac{8}{2} = 4$; $A\,g$ or A n $= \sqrt{4^2 + 3^2} = 5$; and $r\,n = 5 - 3 = 2.$

Weight of the pyramid $= 8^2 \times \frac{12}{3} \times 100 = 25600$ lbs.

$\therefore$ Work $=$ weight $\times\ rn = 25600 \times 2 = 51200$.

Observation. The work necessary to overturn any body is the true measure of its stability.

Ex. 8. A cistern is 12 ft. long, 6 ft. broad, and 8 ft. deep. The height of the bottom of the cistern from the water in the well is 36 ft. It is required to find the work in filling the cistern with water.

Wt. water in the cistern $= 12 \times 6 \times 8 \times 62{\cdot}5 = 36000$ lbs.

Now the height to which the centre of gravity of the water is raised $= \frac{8}{2} + 36 = 40$ ft.

$\therefore$ Work $= 36000 \times 40 = 1440000$.

Ex. 9. Required the same as in the last example, when the cistern is 6 ft. long, 3 ft. broad, and 4 ft. deep, and the bottom of the cistern stands 18 ft. from the water in the well.

Ans. 90000.

WORK IN EXCAVATIONS WHEN THE MATERIAL IS TRANSPORTED BY MEANS OF BARROWS.

13. The following method of calculating the work of excavation is that which is employed by the most eminent French engineers.

The material is classed according to the number of pickmen necessary to keep at work a certain number of shovellers. Thus earth, of mean quality, requires 3 of the former to 2 of the latter; whereas earth, of a harder quality, will require a greater proportion of pickmen. This proportion will, of course, be fixed by the judgment of the engineer. A *relay* is the distance a barrowman will wheel a full barrow and return with an empty one, while a shoveller is filling another barrow. This distance on the horizontal line is calculated to be 120 feet. The barrow is supposed to contain 1 cubic foot of the material, and a shoveller to lift 500 cubit feet of earth per day. When the material is to be raised to the surface, this is supposed to be done by means of *ramps* formed of planks, each 12 feet in length, and having a rise of 1 foot. The length of the relay upon the ramp is 80 feet. The *mean height* to which the material is raised, is the height to which the centre of gravity of the whole material is raised.

The consistency of these views may be shown by the following calculation.

A stout English labourer can fill 500 barrows in a day of 10 hours long. A barrowman upon the level can transport 130 lbs. to the distance of 7800000 feet in a day of 10 hours, whereas upon the ramp he can perform 720 effective units of work per minute in opposition to gravity. Now assuming the weight of a cubic foot of material to be 130 lbs., we have,

No. barrows wheeled on the level per day $= \frac{7800000}{130 \times 120}$ $= 500$.

Effective work in wheeling one barrow on the ramp $= \frac{80}{12} \times 130 = 833$.

$\therefore$ No. barrows wheeled on the ramp per day $= \frac{720 \times 60 \times 10}{833}$ $= 500$ nearly.

This result shows, that the work of a labourer upon the ramp, is very nearly the same as it is on the level plane.

Ex. 1. 60000 cubic feet of earth are to be excavated and then removed to the mean distance of 480 feet, the material is such as to require 3 pickmen to 2 shovellers, it is required to find the workmen of each sort necessary to complete the work in 20 days.

Number relays $= \frac{480}{120} = 4$.

Hence for each shoveller we must have 4 barrowmen; and as there are 3 pickmen for every 2 shovellers, the number of pickmen will be 3 times the $\frac{1}{2}$ of the number of shovellers.

Number Shovellers to do the work in 1 day $= \frac{60000}{500} = 120$.

Barrowmen ,, $= 120 \times 4 = 480$.

Pickmen ,, $= \frac{3}{2}$ of $120 = 180$.

Now since the work is to be done in 20 days, the number of men will be the $\frac{1}{20}$ of the number required to do the work in 1 day.

$\therefore$ Number Shovellers to do the work in 20 days $= \frac{120}{20} = 6$.
Barrowmen ,, $= \frac{480}{20} = 24$.
Pickmen ,, $= \frac{180}{20} = 9$.

Ex. 2. Required the same, as in the last example, when the material = 50000 cubic feet, the mean distance = 240 feet, and the material requires 4 pickmen to 3 shovellers.

Ans. 5 *Shovellers,* $6\frac{2}{3}$ *Pickmen, and* 10 *Barrowmen.*

Ex. 3. What would be the cost of the work in example 1, allowing the pickmen 3*s*., the shovellers 2*s*. 6*d*., and the barrowmen 2*s*. per day?

Wages of shovellers = 2*s*. 6*d*. × 120 = £15.
,, barrowmen = 2*s*. × 480 = £48.
,, pickmen = 3*s*. × 180 = £27.
$\therefore$ Total cost £90.

Ex. 4. Required the cost of excavating and then transporting 40000 cubic feet of material to the mean distance of 700 feet by means of barrows, allowing 3 pickmen to 2 shovellers, and each workman 2*s*. 6*d*. per day.

Number relays $= \frac{700}{120} = 5{\cdot}83$.

As a shoveller lifts 500 feet of earth per day.

The number days' work of Shovellers $= \frac{40000}{500} = 80$.

,, ,, Pickmen $= \frac{80 \times 3}{2} = 120$.

,, Barrowmen $= 80 \times 15{\cdot}83 = 466{\cdot}6'$.

$\therefore$ Total days' work $= 666{\cdot}6$; and cost $= 666{\cdot}6' \times$ 2*s*. 6*d*. = £83. 6*s*. 8*d*.

Ex. 5. Required the same, as in the last example, when the material = 25000 cubic feet, the mean distance 360 feet, and the material requires 2 pickmen to 3 shovellers. *Ans.* £29 3*s*. $4\frac{3}{4}$*d*.

Ex. 6. 70000 cubic feet of earth are first to be raised to the mean height of 8 feet by means of ramps, and then removed to the mean horizontal distance of 480 feet, the material requires 4 pickmen to 3 shovellers; it is required to find the cost of the work allowing each labourer 2*s*. per day.

Here there will be 8 planks in the ramp.

$\therefore$ Length ramp = 12 ft. × 8 = 96 ft.

Number relays on the ramp $= \frac{96}{80} = 1{\cdot}2$.

Number of relays on the level $= \frac{480}{120} = 4$.

$\therefore$ Total number relays $= 1{\cdot}2 + 4 = 5{\cdot}2$.

Days' work of shovellers $= \frac{70000}{500} = 140$.

" barrowmen $= 140 \times 5{\cdot}2 = 728$.

" pickmen $= 140 \times \frac{4}{3} = 186\frac{2}{3}$.

$\therefore$ Total number days' work $= 1054\frac{2}{3}$.

Cost $= 1054\frac{2}{3} \times 2s. = £105.\ 9s.\ 4d.$

Ex. 7. Required the same, as in the last example, when the mean height = 12 feet, the horizontal distance = 300 ft., and the material requires 2 pickmen to 3 shovellers.

Ans. £83. 10*s.* 8*d.*

WORK IN OVERCOMING THE RESISTANCE OF FLUIDS.

14. The resistance of the atmosphere to the motion of the body, varies as the square of the velocity. Thus, if the velocity of the body be increased 2 times, the resistance will be increased 4 times, if the velocity be increased 3 times the resistance will be increased 9 times, and so on;—because when the velocity of the body is increased, there is not only a greater volume of air to be displaced by the moving body, but it also strikes with a greater force upon the fluid particles. The resistance also obviously increases with the extent of the surface opposed to the air.

Ex. 1. What must be the H. P. of the engine in Ex. 1, 8, supposing the resistance of the atmosphere to the whole train to be 33 lbs. when moving at the rate of 10 miles per hour?

Here, $\frac{30}{10} = 3$, that is, the speed is increased 3 times, there fore the resistance of the air will be increased 9 times.

$\therefore$ Resistance air $= 33 \times 9 = 297$ lbs.

Work due to the resistance air per min. $= 297 \times 2640$.

But the work of friction per min. $= 400 \times 2640$.

∴ Total work per min. $= (297 + 400) \times 2640$

And H. P. $= \frac{697 \times 2640}{33000} = 55 \cdot 76$.

Ex. 2. A train of 100 tons moves at the steady speed of 40 miles per hour on a level train; it is required to find the H. P. of the engine taking the resistance of the air as in the last example.

Here the speed per min. $= 3520$.

Work due to friction per min. $= 8 \times 100 \times 3520$.

„ the air „ $= (\frac{40}{10})^2 \times 33 \times 3520$.

∴ Total work per min. $= 4674560$, and H. P. $= 141$.

Ex. 3. Required the H. P. of the engine in Ex. 7, 11, supposing the resistance of the atmosphere to be the same as in the last example.

Here the resistance of the air $= (\frac{50}{10})^2 \times 33 = 25 \times 33$ lbs.

∴ Work due to this resistance per min. $= 25 \times 33 \times 4400 = 3630000$.

But work due to friction and gravity per min. $= 187734$.

∴ H. P. $= \frac{363000 \times 187734}{33000} = 115 \cdot 6$.

Ex. 4. The H. P. of an engine $= 120$, the uniform speed on the level rail $= 30$ miles, the resistances of the air and friction as usual; required the weight of the train.

Here the speed per min. $= 2640$ ft.

Work due to the air per min. $= 3^2 \times 33 \times 2640 = 784080$.

Work engine per min, $= 120 \times 33\,000 = 3960000$.

∴ Work engine which goes to overcome friction $= 3960000 - 784080$.

Work due to friction per min, in moving 1 ton $= 8 \times 2640$.

∴ No. tons $= \frac{3960000 - 784080}{8 \times 2640} = 150 \cdot 3$.

Ex. 5. Required the same as in the last example, when H. P. $= 140$, and the speed $= 50$ miles. *Ans.* 28·1.

Ex. 6. Required the weight of the train, in the last example, when it descends an incline having a rise of $\frac{1}{10}$ in 100. *Ans.* 39.

Ex. 7, If a boat, weighing 15 tons, undergoes a resistance of 33 lbs. when sailing at the rate of 2 miles per hour, what must be the effective H. P. to sustain a speed of 8 miles?

Here, $\frac{8}{2} = 4$, that is the velocity is increased 4 times.

$\therefore$ Resistance of the water $= 4^2 \times 33 = 528$ lbs.

Work of resistance per hour $= 528 \times 8 \times 5280$.

$\therefore$ H. P. $= \frac{528 \times 8 \times 5280}{60 \times 33000} = 11{\cdot}2$.

Ex. 8. How many horses would be required to move the boat, in the last example, at the rate of 5 miles per hour.

Traction $= (\frac{5}{2})^2 \times 33 = 206$ lbs.

By the table Art. 6, the traction of a horse at 5 miles is about 41 lbs.

$\therefore$ No. horses $= \frac{206}{41} = 5$.

Ex. 9. What would be the cost of conveying 1 ton for 1 mile, in the last example, allowing 5*s*. per day, of 6 hours long, for each horse?

Distance moved per day $= 5 \times 6$.

$\therefore$ Cost of 15 tons for 1 mile $= \frac{5 \times 5s.}{5 \times 6}$.

$\therefore$ " 1 ton " $= \frac{5 \times 5}{5 \times 6 \times 15} = \frac{2}{3}d.$

THE MODULUS OF A MACHINE.

15. The modulus* of a machine is the fraction which expresses the relation of the work done to the work applied. Thus if a machine should only perform one-half the work that is applied to it, then the modulus, in this case, would be ½ or ·5. However perfectly a machine may be constructed, there must always be a certain amount of work destroyed by the friction of the parts. The work that is thus destroyed depends upon the extent and nature of the rubbing surfaces. In many machines also, the power of the labouring agent may be exerted in a more or less efficient manner. Hence it is, that

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* Professor Moseley first gave the general theory of the moduli of machines.

the modulus of one machine sometimes differs very much from that of another. The following table of machines for the raising of water is taken from Morin's Mecanique Pratique.

	Modulus.
Incline chain pump	·38
Upright ,,	·53
Bucket wheel	·6
Chinese wheel	·58
Archimedian screw	·7
Pumps for draining mines, &c.	·66

Ex. 1. If 2 H. P. be applied to a bucket wheel, how many cubic feet of water will be raised per hour to the height of 18 feet?

Work applied per hour = $2 \times 33000 \times 60$.
Work done ,, $= 2 \times 33000 \times 60 \times \cdot 6$.
Work in raising 1 ft. of water = $62 \cdot 5 \times 18$.

$$\therefore \text{No. c. ft.} = \frac{2 \times 33000 \times 60 \times \cdot 6}{62 \cdot 5 \times 18} = 2112.$$

Ex. 2. If 5 H. P. be applied to a machine, lifting 3000 feet of water per minute to the height of 40 feet, what is the modulus of the machine?

Work applied per hour = $5 \times 33000 \times 60$.
Work done ,, $= 3000 \times 62 \cdot 5 \times 40$.

$$\therefore \text{Modulus} = \frac{\text{Work done}}{\text{Work applied}} = \cdot 75.$$

Ex. 3. How many H. P. must an engine have to pump 7000 feet of water per hour, from a mine whose depth is 33 fathoms?

Work in raising the water per hour = $7000 \times 62 \cdot 5 \times 33 \times 6$.

Effective work of one H. P. per hour = $33000 \times 60 \times \cdot 66$

$$\therefore \text{H. P.} = \frac{7000 \times 62 \cdot 5 \times 33 \times 6}{33000 \times 60 \times \cdot 66} = 66 \cdot 2.$$

Ex. 4. A cistern is 10 ft. long, 7 ft. broad, and 8 ft. deep. The height of the bottom of the cistern from the water in the well is 56 ft. In what time will a man pump the cistern full of water, allowing that he performs 2600 units of work per minute, and that the modulus of the pump is ·66?

Wt. water in the the cistern = 35000 lbs.

Ht. to which the centre of gravity of the water is raised = $\frac{8}{2}$ + 56 = 60 ft. See Art. 12.

∴ Work = 35000 × 60 = 2100000.

Effective work of the man per h. = 2600 × 60 × ·66 = 102960.

$$\therefore \text{No. hours} = \frac{2100000}{102960} = 20{\cdot}4 \text{ nearly.}$$

Ex. 5. Required the same as in the last example, when the cistern is 7 ft. by 5 ft., and the depth 6 ft., and the height from the water in the well 37 ft.

Ans. 5·1 *hours nearly.*

Ex. 6. What work would the water in the cistern, of Ex. 4, perform upon a wheel, which takes up $\frac{2}{5}$ of the work?

Here wt. water = 35000 lbs.

Space through which the centre of gravity descends = 4 ft.

∴ Work = 35000 × 4 × $\frac{2}{5}$ = 56000.

Ex. 7. In what time will a pumping engine of 4 horse powers fill a tank 15 ft. long, 10 ft. broad, and 6 ft. deep, from a well 20 sq. ft. in the section, supposing the level of the water in the well to be 40 ft. below the bottom of the tank, when the engine began to work, and that no water ran in during the process?

Wt. water raised = 900 × 62·5.

Depth to which the water is depressed in the well = $\frac{900}{20}$ = 45 ft.

Now the distance between the centres of gravity of the water in the tank and that which is in the well is 40 + $\frac{45}{2}$ + $\frac{6}{2}$ = 65·5 ft.

∴ Work in pumping water = 900 × 62·5 × 65·5 = 3684375.

Effective work of the engine per min. = $4 \times 33000 \times \cdot 66 = 87120$.

$\therefore$ No. min. = $\frac{3684375}{87120} = 42\cdot 2$.

Ex. 8. Required the same as in the last example, when the tank is 12 ft. long, 9 ft. broad, and 8 ft. deep. *Ans.* 40·6*min.*

LABOURING FORCES.

16. We shall now consider, more in detail, some of the most important labouring forces. Besides the labouring force of animals, the intellect of man has subjugated the elements of nature to perform work;—the rolling stream and the waterfall have been made to move the water-wheel, in order to grind our corn;—the ocean, agitated by the attraction of distant spheres, does our work in its ceaseless ebb and flow, by carrying our ships and barges to and from our inland cities;—the air does our work, by pressing upon the sails of the windmill, and bearing vessels to our shores laden with the merchandise of distant lands;—fire and water do our work, in the form of steam,—an agent as various as to the mode of its application, as it is exhaustless with respect to the power which it developes;—and the time may not be distant when even the most subtle and terrific agents of nature may be constrained to perform useful labour.

WORK OF WATER.

Work of a Water-wheel.

17. When water, or any body falls from a given height, the work which it is capable of performing, will just be that which would be done upon it in raising it to the height from which it has fallen. It is no matter in what way this work is used,—whether the water falls into the buckets of an overshot wheel, or delivers its work upon the paddles of an undershot wheel,—

the labouring force of the water will always be equal to the work due to the height of the fall.

Ex. 1. The breadth of a stream is 4 feet, depth 3 feet, mean velocity of the water 15 feet per minute, and the height of the fall 20 feet; required the H. P. of the water-wheel which does $\frac{7}{10}$ of the work of the water, and also the number of bushels of corn which the wheel will grind in a day of 10 hours.

Water going over the fall per min. $= 4 \times 3 \times 15 =$ 180 c. ft.

Weight ,, ,, $= 180 \times 62 \cdot 5$ lbs.

As this water descends the height of 20 feet,

The work of the water per min. $= 180 \times 62 \cdot 5 \times 20$.

But the wheel does $\frac{7}{10}$ of this work,

$\therefore$ Work wheel per min. $= 180 \times 62 \cdot 5 \times 20 \times \cdot 7$.

And H. P. $= \frac{180 \times 62 \cdot 5 \times 20 \times \cdot 7}{33000} = 4 \cdot 77$.

Now a horse power is able to grind 1 bushel of corn per hour,

$\therefore$ Number bushels per day $= 10 \times 4 \cdot 77 = 47 \cdot 7$.

Ex. 2. Required the same as in the last example, when the breadth of the stream $=$ 3 ft., depth $=$ 2 ft., and velocity $=$ 12 ft. *Ans. H. P.* $= 1 \cdot 9$, *and* 19 *bush.*

Ex 3. The section of a stream is 4 feet by 2 feet, the mean velocity is 20 feet per minute, and the height of the fall 15; what would be the H. P. of the water-wheel whose modulus is $\cdot 68$, and how many cubic feet of water per min. would the wheel pump from the bottom of the fall to the height of 115 feet?

Content water going over the fall per min. $= 4 \times 2 \times 20$ $= 160$ c. ft.

Weight ,, ,, $= 160 \times 62 \cdot 5$ lbs.

Work of the wheel per min. $= 160 \times 62 \cdot 5 \times 15 \times \cdot 68$.

And H. P. $= \frac{160 \times 62 \cdot 5 \times 15 \times \cdot 68}{33000} = 3 \cdot 09$.

Work in pumping 1 c, ft. of water $= 62 \cdot 5 \times 115$,

∴ Number feet pumped per min. $= \frac{160 \times 62{\cdot}5 \times 15 \times {\cdot}68}{62{\cdot}5 \times 115}$ $= 14{\cdot}2$.

Ex. 4. How many cubic feet of water per minute will the wheel, in the last example, pump from the top of the fall to the elevation there mentioned?

In this case, only a portion of the water of the stream goes upon the wheel.

Space through which the water is pumped $= 115 - 15 =$ 100 ft.

Work in pumping 1 c. ft. $= 62{\cdot}5 \times 100$.

Work of 1 c. ft. water going on the wheel $= 62{\cdot}5 \times 15 \times 68$.

∴ Number ft. going on the wheel to pump 1 ft. $= \frac{62{\cdot}5 \times 100}{62{\cdot}5 \times 15 \times {\cdot}68} = 9{\cdot}8$.

Hence, in order to pump 1 ft. there must 9·8 c. ft. fall upon the wheel, or it takes 10·8 ft. of the stream to pump 1 ft, therefore the water pumped is just $\frac{1}{10{\cdot}8}$ part of the whole. Now the whole water = 160 ft. per minute.

∴ Water pumped per min. $= \frac{160}{10{\cdot}8} = 14{\cdot}8$ c. ft.

Hence it appears that it would be more advantageous to pump water from the *top* of the fall.

Ex. 5. How many cubic feet of water will the wheel in Ex. 1, raise per minute, from the top of the fall to the height of 40 feet above that point? *Ans.* 46·6.

Ex. 6. How many cubic feet of water would be required per minute, to drive a water wheel of 4 effective horse powers, supposing the height of the fall to be 16 feet, and the modulus of the wheel ·68?

Work of the wheel per min. $= 33000 \times 4$.

Effective work of 1 c. ft. of water $= 62{\cdot}5 \times 16 \times {\cdot}68$.

Now as many times as this work of 1 c. ft. can be taken out of the work of the wheel per minute so many feet of water must go over the fall per minute.

∴ Number feet water $= \frac{33000 \times 4}{62{\cdot}5 \times 16 \times {\cdot}68} = 194{\cdot}1$.

Ex. 7. Required the same as in the last example, when the H. P. = 2, and the height of the fall = 20 ft. *Ans.* 77·6.

Ex. 8. What must be the fall of the stream in Ex. 3, so that the wheel may perform the work of 3 horses?

Ans. 14·5 ft.

Work of the Hydraulic Ram.

18. From the experiments of Eytelwein, a celebrated German mathematician, it appears that the modulus of a hydraulic ram of the best construction is ·87.

Ex. 1. In a hydraulic ram there are 14 gallons of water spent per minute, the fall of the water is 10 feet, it is required to determine the number of cubic feet of water which will be raised per minute to the height of 30 feet.

Weight water spent per min. = 140 lbs.

∴ Work applied per min. = 140 × 10 = 1400.

∴ Work done per min. = 1400 × ·87.

Work in raising 1 ft. of water = 62·5 × 30.

$$\therefore \text{ No. ft. water } = \frac{1400 \times \cdot 87}{62\cdot 5 \times 30} = \cdot 65.$$

Ex. 2. Required the same as in the last example, when the water spent = 11 gallons, and the fall = 9 ft. *Ans.* ·46.

Ex. 3. To what height would the ram, in Ex. 1, raise ¼ cubic foot of water per minute? *Ans.* 39*ft.*

Work of the Sun performed by evaporation.

19. The heat of the sun is continually raising water into the atmosphere by evaporation. The vapour that is thus raised forms clouds at considerable elevations above the earth's surface; the rain, therefore, that falls from these clouds may be taken as the measure of the work of evaporation. In the torrid zone the annual fall of rain, at a medium, is about 105 inches, while in the extreme parts of the north temperate zone it is not more than 15 inches. The mean of these quantities will be 5 feet, which may be presumed to be about the mean depth of the rain falling upon the whole of the earth's surface.

Now if we suppose 1000 feet to be the mean height from which rain falls, we have,

The work upon 1 mile in H.P. $= \frac{27878400 \times 5 \times 62{\cdot}5 \times 1000}{365 \times 24 \times 60 \times 33000}$ $= 502.$

$\therefore$ Work on the whole of the earth's surface = 502 H. P. × 197000000 = 98894000000 H. P.

Now, Arago states that all the steam engines in Great Britain are equivalent to about 1 million of horse powers; hence it follows, that the work due to the sun's evaporation is more than 98894 times the work of all the steam engines in this country. Such is the stupendous scale upon which the operations of nature are conducted. Although but a very small part of this work is available for the purposes of machinery, yet it all serves an important end in the vast mechanism of creation. In fact, as we raise water by our machines to revive the drooping vegetation on some little arid spot of the earth, so nature, or rather the beneficent God of nature, by the process of evaporation, scatters refreshing showers over every portion of the globe.

Work of a Tide upon the Thames.

20. A spring tide raises the level of the water at London bridge, upon an average, 18 ft. 6 in., and at Battersea bridge 11 ft. 6 in., the mean height, therefore, between these places is 15 ft. Taking 900 ft. as the mean breadth of the river, and the distance between Battersea and London bridge to be 5 miles, we have,

Wt. water elevated each tide between these places $= 900 \times 15 \times 5 \times 5280 \times 62{\cdot}5.$

Now the work done in each tide, by Art. 12, will be this weight multiplied by the height the centre of gravity of the water is elevated, which, in this case, is 7·5 feet.

$$\therefore \text{H.P.} = \frac{900 \times 15 \times 5 \times 5280 \times 62{\cdot}5 \times 7{\cdot}5}{12{\cdot}5 \times 60 \times 33000} = 6750.$$

The work, for this distance, will be equivalent to the work of 67 engines of 100 horse powers. And the work for the whole river will *considerably* exceed the work of 670 engines of 100 horse powers.

Work of the River Niagara.

21. This river has a total descent of 334 feet, and discharges about 40 millions of tons of water per hour; hence we have,

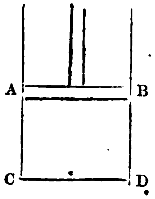
$$\text{Wt. water discharged per min.} = \frac{40000000 \times 2240}{60} \text{ lbs.}$$

$$\therefore \text{H. P.} = \frac{40000000 \times 2240 \times 334}{60 \times 33000} = 15114343.$$

This river, therefore, (see Art 19) is capable of doing more work than 15 times the work of all the steam engines in Great Britain.

WORK OF STEAM.

Work of Steam, having a mean pressure.



22. If steam in the cylinder A D exert any constant, or mean, pressure upon the piston A B, say of 30 lbs. per square inch, then if a weight of 30 lbs. be placed upon every inch of surface in the piston, the elastic vapour would just be able to move the piston with its weights through the length of the stroke in opposition to gravity; therefore the work performed upon 1 inch of the piston in 1 stroke will be the pressure of the steam upon 1 inch multiplied by the number of feet in the stroke, and the work upon the whole piston will be the work upon 1 inch multiplied by the number of inches in the whole piston.

In the high pressure engine, the pressure of the atmosphere,—about 15 lbs. per sq. inch,—is opposed to the pressure of the steam. Besides this, a considerable portion of the pressure of the steam is required to overcome the friction of the parts of the engine. As a mean estimate, 1 lb. to the sq. inch is allowed for the friction due to the engine when unloaded; and an additional friction of $\frac{1}{7}$ the effective, or useful, pressure for the resistance necessary to overcome the friction of the loaded engine. Thus, if the pressure of the steam is 50 lbs., we shall have 15 lbs. for the pressure of the air, and 1 lb. for the resistance of the friction of the unloaded piston, and then the remaining 34 lbs. will be taken up by the useful load and the friction arising from that load, that is, load + $\frac{1}{7}$ load = 34 lbs., or $\frac{8}{7}$ load = 34 lbs., and therefore load = 28 lbs. This load is called the effective pressure of the steam,

Ex. 1. The area of the piston of a steam engine is 2000 square inches, the mean effective pressure of the steam 15 lbs., the length of the stroke 8 feet, the number of strokes performed per minute 20; required the H. P.

Work done upon 1 inch of the piston in 1 stroke = $15 \times 8 = 120$.

Work upon the whole piston in 1 stroke = 120×2000.

,, ,, 20 strokes = $120 \times 2000 \times 20$.

As this is also the work done per minute, we have,

$$\text{H. P.} = \frac{120 \times 2000 \times 20}{33000} = 145 \cdot 4.$$

Ex. 2. The area of the piston of a high pressure engine is 500 inches, the length of the stroke 6 ft., the pressure of the steam 40 lbs., and the number of strokes made per min. 16; it is required to find the number of feet of water which the engine will pump per min. from a mine whose depth is 80 fathoms, making the usual allowance for friction and the modulus of the pump.

Here, load + $\frac{1}{7}$ load + 1 + 15 = 40; ∴ $\frac{8}{7}$ load = 24; and load = 21 lbs.

∴ Useful work of engine per min. = $21 \times 500 \times 6 \times 16 \times \cdot 66 = 665280$.

Work in pumping 1 ft. of water = $62 \cdot 5 \times 80 \times 6 = 30000$.

∴ No. c. ft. = $\frac{665280}{30000} = 22 \cdot 1$.

Ex. 3. Required the same as in the last example, when the pressure of the steam is 48 lbs., and the depth of the mine is 160 fathoms. *Ans* 14·7.

Ex. 4. Required the same as in Ex. 1, when the area of the piston = 500 inches, pressure of the steam = 33 lbs., length of the stroke = 4 feet, and the number of strokes per min. = 16. *Ans.* 32.

Ex. 5. What must be the mean effective pressure of the steam in the last example, so that the engine may do the work of 20 horses?

Work engine per min. = 20 × 33000.

Work of 1 lb. pressure of steam per min. = 500 × 4 × 16.

$\therefore$ No lbs. $= \dfrac{20 \times 33000}{500 \times 4 \times 16} = 20\tfrac{5}{8}$.

Ex. 6. The area of the piston of a high pressure engine is 3000 inches, the length of the stroke 10 feet, the number of strokes per min. 16; required the mean pressure of the steam so that the engine may perform the work of 120 horses, making the usual allowance for friction.

Work per min. of steam with 1 lb. effective pressure = 3000 × 1 × 10 × 16.

Effective work per min. = 33000 × 120.

$\therefore$ Effective pressure on 1 inch of the piston $= \dfrac{33000 \times 120}{3000 \times 10 \times 16}$ = 8·25 lbs.

$\therefore$ Pressure steam $= 1 + 15 + 8{\cdot}25 + \tfrac{1}{7} \times 8{\cdot}25 =$ 25·4 lbs.

Ex. 7. Required the same as in the last example, when the area of the piston = 800 inches, the length of the stroke = 6 ft., the number of strokes per min. = 20, and the H. P. = 40. *Ans.* 31·7 lbs.

Ex. 8. The length of the stroke of a high pressure engine is 8 feet, the area of the piston 1000 inches, and the number of strokes made per min. 20; what must be the pressure of the steam so that the engine may pump 80 cubic feet of water per min. from a mine whose depth is 120 fathoms, making the usual allowance for friction and the modulus of the pump, Work done per min. $= 80 \times 62{\cdot}5 \times 120 \times 6 = 3600000$.

$\therefore$ Work engine per min. $\times \cdot 66 = 3600000$.

And work engine per min. $= 5454545$.

Useful work on 1 inch of the piston in 1 stroke $= \frac{5454545}{20 \times 1000} = 272$.

$\therefore$ Useful load $= \frac{272}{8} = 34$ lbs.

Pressure steam $= 34 + \frac{34}{7} + 15 + 1 = 54{\cdot}8$ lbs.

Ex. 9. Required the same as in the last example, when the depth of the mine is 100 fathoms, and the length of the stroke 10 feet. *Ans.* 41·9 *lbs.*

Work of Steam, with a mean pressure, considered in relation to the water evaporated.

23. The true source of work in the steam engine is the evaporating power of the boiler. The magnitude of the work not only depends upon the quantity of water evaporated, in a given time, but also upon the temperature, and consequently the pressure at which the steam is formed. Experimental tables have been formed, giving the relation of the volume and pressure of steam raised from a cubic foot of water;* these tables will enable us to find the volume of the steam when its pressure and the volume of the water are given, and *vice versa.* The following will serve as a specimen of this table.

Volume of a cubic foot of water in the form of steam at the corresponding pressures.

Press. lbs.	Vol.	Press. lbs.	Vol.
30	883	48	575
35	767	50	554
40	679	55	508
45	610	60	470

Ex. 1. In a high pressure engine the area of the piston is 100 inches, the length of the stroke 2 ft. 6 in., the effective evaporation of the boiler ·335 c. ft. per minute, the pressure of the steam in the cylinder 50 lbs., and the loss due to friction as explained in Art. 22; required the useful load per square inch of the piston, and the useful H. P.

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* The Author has given a general formula expressing this relation with remarkable exactness, between the range of 3 and 200 lbs. pressure. See Mr. Hann's able work on the steam engine.

Here by Art. 22, $\frac{8}{7}$ useful load $= 50 - 15 - 1 = 34$ lbs.

∴ Useful load $= 29{\cdot}75$ lbs.

Now by the experimental table, a cubic foot of water in the state of steam at 50 lbs. pressure has a volume of 554 c. ft.

∴ Volume steam evapo. per min. $= 554 \times {\cdot}335$.

Volume discharged at each stroke $= \frac{100 \times 2{\cdot}5}{144} = 1{\cdot}73$.

∴ Number strokes per min. $= \frac{554 \times {\cdot}335}{1{\cdot}73} = 107$.

Work in 1 stroke $= 29{\cdot}75 \times 100 \times 2{\cdot}5$.

Work in 1 min. $= 29{\cdot}75 \times 100 \times 2{\cdot}5 \times 107$.

And H. P. $= \frac{29{\cdot}75 \times 100 \times 2{\cdot}5 \times 107}{33000} = 24$.

Ex. 2. Required the H. P., as in the last example, when the area of the piston = 120 inches, the length of the stroke = 2 ft. 3 in., the evaporation = ·4 ft. and the pressure of the steam = 48 lbs. *Ans.* 28.

Ex. 3. The area of the piston of a high pressure engine is 144 inches, the length of the stroke 3 feet, the pressure of the steam in the cylinder 48 lbs., the number of strokes per min. 20, and the loss of friction as usual; it is required to find the useful load, the water evaporated per hour, and the useful H. P. of the engine.

Useful load + $\frac{1}{7}$ useful load $= 48 - 15 - 1 = 32$ lbs.

∴ Useful load $= 28$ lbs.

Volume steam discharged per min. $= \frac{144 \times 3}{144} \times 20 = 60$. cubic feet.

∴ „ „ hour $= 3600$.

By the experimental table 1 c. ft. of water yields 575 c. ft. of steam at 48 lbs. pressure.

∴ Cubic ft. water evapo. per hour $= \frac{3600}{575} = 6{\cdot}2$.

H. P. $= \frac{144 \times 28 \times 3 \times 20}{33000} = 7{\cdot}3$.

Ex 4. Required the same as in the last example when the area of the piston = 96 inches, length of the stroke = 2½ ft.

pressure of the steam = 60 lbs., and the number strokes = 100. *Ans.* 21·2 *ft., and H. P.* = 28.

Ex. 5. Required the duty of the engine in Ex. 3, allowing that 1 bushel of coals can evaporate 11·5 feet of water.

Here the useful work per hour = 144 × 28 × 3 × 20 × 60.

Now this work is done by 6·2 ft. of water.

∴ Work of 11·5 ft. of water, or 1 bus. coals = 144 × 28 × 3 × 20 × 60 × $\frac{11 \cdot 5}{6 \cdot 2}$ = 27 millions nearly, which is called the duty of the engine.

Observation. It has been found, by experiment, that whatever may be the pressure at which the steam is formed, the quantity of fuel necessary to evaporate a given volume of water is always the same. Hence it follows that it is most advantageous to employ steam of a high pressure.

Ex. 6. A train of 100 tons moves at the uniform speed of 20 miles per hour upon a level rail, the resistance of friction upon the rail is 7 lbs. per ton, the resistance of the atmosphere 33 lbs. upon the whole train, when the speed is 10 miles per hour, the diameter of the driving wheel 5 feet, the area of the piston 110 inches, the length of the stroke $\frac{4}{3}$ feet, and in addition to the resistances of the preceding questions, the resistance due to the blast pipe is 1·75 lbs. per inch of the piston when the speed of the train is 10 miles per hour. It is required to determine the pressure of the steam, the evaporation of the boiler, and the number of bushels of coals necessary for a journey of 200 miles, allowing that 1 bushel will evaporate 11·5 c. ft. of water.

Total resistance to the motion of the carriages = 7 × 100 + $(\frac{20}{10})^2$ × 33 = 832 lbs.

Space moved over in 1 revo. of the driving wheel = 5 × 3·1416.

∴ Work in 1 revo. = 5 × 3·1416 × 832.

Work of 1 lb. pressure upon 1 inch of the pistons in 1 revo. of the driving wheel = 1 × 110 × $\frac{4}{3}$ × 4. It will here be observed, that the engine has two cylinders, and that each piston makes two strokes while the driving wheel turns round once.

∴ Effective pressure on 1 inch of the piston $= \frac{5 \times 3{\cdot}1416 \times 832}{110 \times \frac{4}{3} \times 4} = 22{\cdot}27$.

It has been found by experiment, that the resistance of the blast pipe increases with the speed of the engine.

∴ Resistance due to the blast pipe $= 1{\cdot}75 \times \frac{20}{10} = 3{\cdot}5$ lbs.

∴ Total pressure steam on the piston $= 22{\cdot}27 + \frac{1}{7} \times 22{\cdot}27 + 3{\cdot}5 + 1 + 15 = 45$ lbs. nearly.

Number revo. driving wheel per min. $= \frac{20 \times 5280}{60 \times 5 \times 3{\cdot}1416} = 112$.

∴ Number of strokes of the piston per min. $= 112 \times 4 = 448$.

∴ Volume steam discharged per min. $= \frac{110}{144} \times \frac{4}{3} \times 448 = 456$ ft.

But from the experimental tables 1 foot of water produces 610 feet of steam at 45 lbs. pressure.

∴ Number feet water evapo. per min. $= \frac{456}{610} = {\cdot}74$.

As 1 bushel of coals evaporates 11·5 feet of water,

The number bus. of coals used per min. $= \frac{{\cdot}74}{11{\cdot}5}$.

∴ Number bus. for 10 hours, or the distance of 200 miles $= \frac{{\cdot}74}{11{\cdot}5} \times 60 \times 10 = 38{\cdot}6$.

Ex. 7. Required the same as in the last example, when the speed = 40 miles, diameter of the driving wheel = 6 feet, area piston = 120 inches, and the length of the stroke = 1·5 feet. *Ans. Pressure* = 60 *lbs, nearly;* 1·98 *ft. of water per min.*; 51 *bushels.*

Ex. 8. Supposing the weight of the useful load, moved in the last example to be 40 tons, what would be the cost of the transport of 1 ton per mile, due to the expenditure of fuel, allowing the coals to be 15*d.* per bushel?

Ans. about $\frac{1}{3}$ *of a farthing.*

Ex. 9. In a locomotive engine the area of the piston is 90 inches, the length of the stroke 16 inches, the pressure of the steam 50 lbs., the effective evaporation of the boiler ·7 cubic feet, per minute, the diameter of the driving wheel 5 feet; required the speed of the train per hour

At 50 lbs. pressure 1 foot of water forms 554 feet of steam.

$\therefore$ Volume steam formed per min. $= 554 \times \cdot 7 = 387 \cdot 8$ ft.

Volume steam discharged in 1 revo. of the driving wheel $= \frac{90 \times 16}{1728} \times 4 = 3 \cdot 33$ c. ft.

$\therefore$ Number revo. of wheel per min. $= \frac{387 \cdot 8}{3 \cdot 33} = 116 \cdot 3$.

$\therefore$ Space moved over by the carriage per min. $= 5 \times 3 \cdot 1416 \times 116 \cdot 3$ ft.

And space moved over per hour $= \frac{5 \times 3 \cdot 1416 \times 116 \cdot 3 \times 60}{5280} = 20 \cdot 7$ miles.

Ex. 10. Required the speed, in the last example, when the pressure of the steam is 60 lbs., and the diameter of the driving wheel 6 feet. *Ans.* 21·1 *miles.*

Ex. 11. The area of the piston of a locomotive is 80 inches, the length of the stroke is 15 inches, the pressure of the steam 48 lbs., and the diameter of the driving wheel 5 ft.; required the effective evaporation of the boiler, so that the train may have a speed of 30 miles per hour; required also the effective H. P. of the engine, and the weight of the train, taking the resistances to the motion of the piston the same as in Ex. 6.

No. revo. of the wheel per min. $= \frac{30 \times 5280}{60 \times 5 \times 3 \cdot 1416} = 168$.

$\therefore$ No. strokes of the piston per min. $= 168 \times 4 = 672$.

Load $+ \frac{1}{7}$ load $= 48 - 15 - 1 - 1 \cdot 75 \times 3 = 26 \cdot 75$.

$\therefore$ Load $= \frac{26 \cdot 75 \times 7}{8} = 23 \cdot 4$ lbs.

$\therefore$ Effective work per min. $= 23 \cdot 4 \times 80 \times \frac{5}{4} \times 672 = 1572480$,

And effective H. P. $= \frac{1572480}{33000} = 47$.

Now this effective work of the engine has to support a speed of 30 miles per hour, or 2640 feet per minute, in the train in opposition to the resistances of friction and the atmosphere.

Work due to the resistance of the atmosphere per min. $= (\frac{30}{10})^2 \times 33 \times 2640 = 784080$.

∴ Work due to friction per min. $= 1572480 - 784080 = 788400$.

Work of friction when the train is 1 ton $= 7 \times 2640$.

∴ No. tons in the train $= \frac{788400}{7 \times 2640} = 42{\cdot}6$.

Vol. steam disc. each stroke $= \frac{80 \times 1{\cdot}25}{144}$.

∴ „ „ per min. $= \frac{80 \times 1{\cdot}25 \times 672}{144}$.

But, by the table, a cubic foot of water yields 575 feet of steam.

∴ No. ft. of water evapo. per min. $= \frac{80 \times 1{\cdot}25 \times 672}{144 \times 575} = {\cdot}81$.

Ex. 12. Required the same as in the last example, when the area of the piston is 120 inches, the pressure of the steam 60 lbs., and the speed 50 miles.

Ans. Evaporation = 2·48 ft., H. P. = 157, and weight of the train = 50·3 tons.

Ex. 13. If the engine in Ex. 11, move the train of Ex. 4, art. 11, what must be the effective evaporation of the boiler, and the duty of the engine?

Speed per min. $= \frac{18{\cdot}4 \times 5280}{60} = 1620$ ft. nearly.

Number strokes of the piston $= \frac{1620}{5 \times 3{\cdot}1416} \times 4 = 412$.

Effective work of engine per min. $= 40 \times 33000$.

Work of 1 lb. effective pressure per min. $= 1 \times 80 \times \frac{15}{12} \times 412$.

∴ Effective pressure of steam $= \frac{40 \times 33000}{80 \times \frac{15}{12} \times 412} = 32$.

∴ Pressure steam $= 32 + \frac{1}{7} \times 32 + 15 + 1 + \frac{18{\cdot}4}{10} \times 1{\cdot}75 = 55{\cdot}7$ lbs.

The volume 1 c. ft. of water in the form of steam at 55·7 lbs. pressure is 504 c. ft.

Number c. ft. steam discharged per min. $= \frac{80 \times 15}{1728} \times 412 = 287$.

∴ Number c. ft. water evapo. per min. $= \frac{287}{504} = {\cdot}57$.

Now this water performs 40 × 33000 units of work.

∴ Work 11·5 ft water, or 1 bus. of coals $= 40 \times 33000 \times \frac{11\cdot5}{\cdot57} = 27$ millions, which is the duty of the engine.

Ex 13. Required the duty of the engine in Ex. 1, and the cost of a horse power in a day of 6 hours, arising from the expenditure of fuel, allowing that a bushel of coals cost 4*d.*

Ans. 27½ *millions nearly, and cost* 1⅛*d.*

Work developed by the Condensation of Steam.

24. When water is raised into steam at the boiling temperature, or 212°, its volume is increased 1711 times, or a cubic inch of water will very nearly form a cubic foot of steam. Now if steam at this temperature be allowed to enter the lower part of the cylinder,—(See figure to art. 22,)—then the pressure beneath the piston will just counterpoise the pressure of the air upon the piston, and a small additional force will cause the piston to rise. If then the steam be condensed by a jet of cold water, a vacuum will be formed, and the piston will be pressed downwards with the whole weight of the atmosphere resting upon the surface of the piston. But it has been found that a perfect vacuum cannot be formed in this way, because water gives off vapour at all temperatures. Thus, at the temperature of 150°, the pressure of the vapour is 4 lbs. Now if 14·7 lbs. be taken as the mean pressure of the atmosphere, upon 1 inch of surface, we shall have, by the condensation of steam, upon an average, an effective pressure of 14·7 lbs. — 4 lbs. = 10·7 lbs. upon each inch of the piston.

Ex. 1. Required the work developed by the condensation of a cubic foot of water, supposing 4 lbs. to be the elasticity of the vapour after condensation; required also the duty of the atmospheric engine using the steam in this manner.

Volume of the vacuum formed by condensation = 1710 c. ft.

Pressure on 1 inch of the piston = 14·7 — 4 = 10·7 lbs.

Now if we suppose the area of the piston to be 1 sq. foot, the length of the stroke will be 1710 feet,

$\therefore$ Work of 1 c. ft. water $= 144 \times 10{\cdot}7 \times 1710 = 2634768$.

And duty or work of 1 bus. coals $= 2634768 \times 11{\cdot}5 =$ 30 millions.

Ex. 2. What must be the effective evaporation of the boiler of an atmospheric engine, so that the horse powers may be 30, allowing 3 lbs. for the elasticity of the vapour in the condenser.

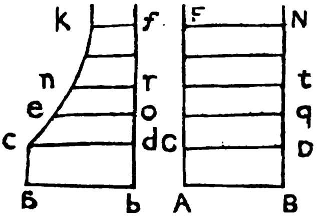
Here work of the engine per min. $= 30 \times 33000$.

Work of 1 c. ft. of water $= (14{\cdot}7 - 3) \times 144 \times 1710 = 2881008$.

$\therefore$ Number c. ft. of water evapo. per min. $= \dfrac{30 \times 33000}{2881008}$ $= {\cdot}34$.

Work of Steam used expansively.

25. When steam is used expansively, it is allowed to enter the cylinder for only a part of the stroke, and then, for the remaining portion, the piston is moved by the expansive force of the steam. This is the most economical way of employing steam power; for all the available work is taken out of the elastic vapour before it is condensed. Now when the volume of steam,—or any elastic fluid,—is increased, its elasticity or pressure is decreased in the same ratio; that is, if its volume is increased two times its pressure will be one half of what it was at first, and so on. This is called Marriotte's law. Let the steam be cut off when the piston is at C D, and let the remaining part of the stroke be divided into any even number of parts; then the pressure of the steam upon the piston when it arrives at the different lines, forming the division, may be ascertained by the law just explained. Let *d c*, *e o*, *r n*, &c. be lines containing as many units as there are units of pressure on the piston at the corresponding points



$\therefore$ Load + $\frac{1}{7}$ load + 1 + 4 = 14·9; hence load = 8·66 lbs.

One cubic foot of water gives rise to 883 cubic feet of steam at 30 lbs. pressure.

$\therefore$ Volume steam discharged per min. = ·2 × 883 = 176·6 ft.

Volume discharged at each stroke = $\frac{1440}{144}$ × 1 = 10 ft.

$\therefore$ Number strokes piston per min. = $\frac{176·6}{10}$ = 17·66.

Now the useful work per min. will be the continued product of the load, the area of the piston, the length of the stroke, and the number of strokes made per min.

$\therefore$ Effective H. P. = $\frac{8·66 \times 1440 \times 4·75 \times 17·66}{33000}$ = 31·7.

Ex. 5. Required the H. P., as in the last example, when the area of the piston = 2000 inches, the length of the stroke = 10 feet, the clearance being neglected, the point at which the steam is cut off = 2 feet, the pressure of the steam = 50 lbs., and the effective evaporation of the boiler = ·8 feet.

Ans. Useful H. P. = 179·2.

This result is obtained by dividing the space of expansion into 4 equal parts.

Ex. 6. What must be the useful load when the length of the stroke is 10 ft., the pressure of the steam 60 lbs., the point at which the steam is cut off 1⅔ ft., and the useless resistances as given in Art. 22? *Ans.* 20·2 *lbs.*

This result is obtained by dividing the space of expansion into four equal parts.

Ex. 7. What must be the evaporation of the boiler in the engine of Ex. 4, when the steam has a pressure of 48 lbs., so that the effective H. P. may be 40.

Proceeding as in Ex. 4, we find, the load = 16·5 lbs.

$\therefore$ Effective work in 1 stroke = 16·5 × 1440 × 4·75.

" " per min. = 40 × 33000.

$\therefore$ Number strokes per min. = $\frac{40 \times 33000}{16·5 \times 1440 \times 4·75}$ = 11·4.

Volume of steam discharged in 1 stroke = 10 ft.

" " per min. = 10 × 11·4 = 114 ft.

Volume 1 c. ft. water in the form of steam at 48 lbs. = 575.
∴ Cubic. ft. water evapo. per min. = $\frac{114}{575}$ = ·2.
Ex. 8. Required the duty of the engine in Ex. 4.
Useful work per min. = 31·7 × 33000.
Now this work is done by the evaporation of ·2 ft. of water
∴ Work of 11·5 ft. of water, or 1 bus. of coals = 31·7 × 33000 × $\frac{11\cdot5}{\cdot2}$ = 60 millions, which is the duty of the engine.

To find the point of the stroke at which the steam must be cut off so as to perform the greatest amount of work.

26. When all the useful work is taken out of the steam, its pressure, at the end of the stroke, must just be equal to the pressure of the useless resistances referred to one inch of the piston; that is, the pressure at the end of the stroke (by Art. 22) = 1 + 4 + $\frac{1}{7}$ load. The load being determined approximately, it then remains for us to find the point at which the steam must be cut off, so as to have its pressure, at the end of the stroke, equal to this quantity.

It will readily be seen, that if the pressure of the steam, at the end of the stroke were greater than the sum of the useless resistances, then its expansion might be carried further, and still be doing useful work; and on the other hand, if its pressure, at the end of the stroke, were less than the sum of the useless resistances, then the expansion would be carried too far, for the steam would be doing less work than that which would be destroyed by the useless resistances.

In our best constructed engines, working with a crank, the clearance has been reduced to a very small fraction: hence in calculations of this kind, this element may be neglected without incurring any sensible error.

Ex. The length of the stroke is 10 ft., and the pressure of the steam is 60 lbs.; required the point at which the steam must be cut off to yield all its work.

In no case can the friction, arising from the load be less than 1 lb.: hence the least value for the pressure of the steam at the end of the stroke will be $1 + 4 + 1 = 6$ lbs. Assuming this to be the pressure, in order to obtain an approximate value for the load, and putting h for the point of the stroke at which the steam is cut off, we have, by Marriotte's law, $60 \times h = 6 \times 10$, $\therefore h = 1$ ft. Hence the portion of the stroke through which the steam acts expansively $= 10 - 1 = 9$ ft. Dividing this space into four equal parts, and proceeding as in Ex. 1, Art. 25, we find,

Work of expansion $= 144 \cdot 3$.

Work done before the steam is cut off $= 60$.

$\therefore$ Total work of steam on 1 inch $= 144 \cdot 3 + 60 = 204 \cdot 3$.

And mean press. steam $= \frac{204 \cdot 3}{10} = 20 \cdot 43$.

By Art. 22, load $+ \frac{1}{7}$ load $+ 5 = 20 \cdot 43$, $\therefore$ load $=$ 13·5 lbs.

Having thus obtained an approximate value for the load, it will enable us to find, more correctly, the point at which the steam must be cut off.

Useless resistances $= \frac{13 \cdot 5}{7} + 5 = 7$ lbs. nearly.

$\therefore 60 \times h = 7 \times 10$, and $h = \frac{7}{6}$ ft.

Going over the whole calculation (after the manner just given) with this more correct datum, we find the load $= 15 \cdot 3$ lbs., and the useless resistances $= 7 \cdot 2$ lbs. nearly.

$\therefore 60 \times h = 7 \cdot 2 \times 10$, and $h = 1 \cdot 2$ ft.

Proceeding in this way the point may be determined to any degree of exactness.*

TRANSMISSION OF WORK BY SIMPLE MACHINES.

27. The object of machinery, properly so called, is to regulate the distribution, or change the direction of work,—not to increase it. If there were no friction or any other resistances

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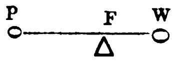
* The Author first gave this method in the Mechanics' Magazine for 1844.

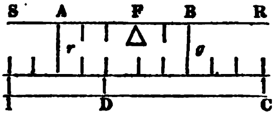
to the motion of the pieces composing a machine, the work that would be given out would be exactly equal to the work applied. Dead matter, by its gravity, produces pressure, and by the intervention of mechanism that pressure may be increased or decreased, but *work* is peculiarly the production of active or living agents. To suppose that machines are capable of augmenting work, would be endowing inert matter with a creative power,—the power of creating work. It is true that man, in a certain sense, creates work by mechanical combinations, when he turns the mountain stream to drive a wheel, or to form a canal; but here, the true source of the work is the heat of the sun. In all instances of labour, performed by inanimate matter, there is some active agent of nature, such as heat, electricity, or gravitation, which gives rise to the work; but, in the case of merely mechanical arrangements, the inert matter is the passive recipient of work, or the channel through which it flows: hence we may lay it down as a fundamental axiom in mechanics, that (abstracted from friction and resistance of the air) *the work done by any machine is the same as the work applied.* Now as the work is the product of pressure and motion, it follows that if the working point of a machine moves more slowly than the driving point, then the pressure at the former will be greater than it is at the latter.

The Lever.

28. Let P W be a rod, or lever, turning upon the fulcrum or centre F; P and W are weights which balance each other; or maintain the lever in equilibrium; then when this is the case, $P \times PF = W \times WF$, that is the units of weight in P multiplied by the units in its distance from F, will be equal to the units of weight in W multiplied by the units in its distance from F.



Here P is called the power, and W the weight or resistance, and a mechanical advantage is gained, inasmuch as a small weight, or pressure, is used to balance a large one. The product of any weight by its distance from the centre of motion, or fulcrum, is called the moment of that weight, and, therefore, when the sum of the moments tending to turn the lever in one direction, is equal to the sum of the moments tending to turn the lever in the opposite direction, then the lever will be in equilibrium. This is a particular case of the principle of the equality of moments.



Proof. Suppose two uniform bars, I D and D C, to be suspended from their centres r and g, by means of cords attached to the points A and B of the lever S R, turning upon the fulcrum F, which must evidently be in the middle of R S, or over the middle of I C, in order to secure equilibrium. Let the weight of the bar I D = 4 lbs., and the weight of D C = 6 lbs. then I D will contain four units of length, and D C six of these units. Now it is obvious, from the figure, that, A F, the distance at which I D acts from the fulcrum will contain 3 units, and B F the distance at which D C acts from the fulcrum will contain 2 units; then since it appears that a weight of 4 lbs., supended at A, balances a weight of 6 lbs., suspended at B, we therefore have the following relation when equilibrium takes place.

$4 \text{ lbs.} \times 3 = 6 \text{ lbs.} \times 2$, that is,

Wt. at A × A F = wt. at B × B F.

In precisely the same way the proposition may be established for any other case.

Levers are divided into three kinds. 1°. When the power and weight are on opposite sides of the fulcrum, as in the case just described. 2°. When the weight is between the fulcrum and the power. 3° When the power is between the fulcrum and the weight. The principle of the equality of moments, obviously, applies to all these cases.

A crow bar, a claw hammer, the handle of a common pump, a pair of scissors, a fire poker, &c., belong to the first kind of lever. And in all these instances a mechanical advantage is given to the power, whenever the arm by which it acts is longer than the arm by which the weight acts. The oar, nut-crackers, the bread knife, the rod of a safety valve, (See fig. Art. 31), &c., belong to the second kind of lever. And, in all these instances, a mechanical advantage is given to the power, because it acts at a greater distance from the fulcrum, or centre of motion, than the weight or resistance. The fire tongs, the sheep shears, the limbs of animals, &c., are instances of the third kind of lever. Here the power must be greater than the weight or resistance, because the former acts at a less distance from the fulcrum than the latter.

Ex. 1. Let $W = 80$ lbs., $W F = 3$ inches, and $P F = 16$ inches; required P. See fig. page 53.

Here by the equality of moments,

$P \times 16 = 80 \times 3, \therefore P = \frac{240}{16} = 15$ lbs.

Ex. 2. Let $W = 90$ lbs., $W F = 4$ inches, and $P = 15$ lbs.; required P F.

$15 \times P F = 90 \times 4, \therefore P F = 24$ inches.

Ex. 3. A man exerts a pressure of 80 lbs. upon a crow-bar at a distance of 4 ft. from the fulcrum, what weight will he balance at the distance of 4 inches from the fulcrum?

$W \times 4 = 80 \times 48, \therefore W = 960$ lbs.

Ex. 4. Required the same as in the last example, when the pressure of 60 lbs. is exerted at the distance of 6 ft. from the fulcrum, and the weight is at 3 inches. *Ans.* 1440*lbs.*

Ex. 5. What is the mechanical advantage of the lever in the last example? *Ans.* 24 ? because the weight is 24 times the power.

Ex. 6. In a lever of the second kind the power acts at 5 ft., and the weight at 2 in. from the fulcrum; required the power necessary to balance a weight of 4 cwts. *Ans.* 14·9*lbs.*

Ex. 7. At what distance from the fulcrum must the pressure, in Ex. 3, be applied, so as to support a weight of 5 cwts.? *Ans.* 28*inches.*

Ex. 8. In a combination of three levers of the first kind, the long arms are 2, 3, and 5 ft. respectively, and the short arms 4, 6, and 2 inches respectively; if a pressure of 8 lbs. be applied, what weight will be balanced? *Ans.* 3·8*tons.*

Ex. 9. A weight of 4 lbs. is placed at the distance of 3 inches from the fulcrum, and another weight of 7 lbs. at the distance of 5 inches: where must a pressure of 3 lbs. be applied to balance the lever? *Ans.* $15\frac{2}{3}$ *inches from the fulcrum.*

Ex. 10. Let P F and W F be the arms of a false balance, a certain weight Q weighs 16 lbs. when put into the scale attached to the long arm, and only 9 lbs. when put into the opposite one; required the true weight of Q.

Here by the equality of movements, we obtain two equations.

$Q \times PF = 16 \times WF$; $Q \times WF = 9 \times PF$.

Multiplying these equations together, and then striking out the common factors, we have,

$Q^2 = 16 \times 9$, $\therefore Q = 12$ lbs.

The principle of Work as applied to the Lever.

29. Let $PF = 5$ ft., $WF = 1$ ft., and $P = 2$ lbs., required W.

Now in order to raise W one foot, P must descend 5 feet, because P F is 5 times W F, hence we have,

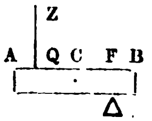
Work of $P = 2 \times 5$; Work of $W = W \times 1$.

$\therefore W \times 1 = 2 \times 5$, and $W = 10$ lbs.

This result is evidently the same as that which would be obtained by the preceding method. Assuming the equality of moments, we may readily establish the principle of the equality of work, and conversely.

The Lever when its weight is taken into account.

30. The tendency of a uniform beam A B to turn upon the fulcrum F, is just the same as if the whole of its weight were collected in its middle point, or centre of gravity C.



For if the preponderating side, A F, be hung from a cord Z Q placed so that $AQ = FB$, then this cord will sustain one half of the weight of the beam.

∴ The moment of the beam $= \frac{1}{2}$ wt. beam × Q F = wt. beam × C F.

Ex. 1. Let a lever A B weigh 4 lbs., A F = 60 inches, F B = 2 inches, a weight W suspended from B = 180 lbs.; required the pressure, P, which must be applied at A to maintain the lever in equilibrium.

Here the weight of the lever acting in its centre of gravity C, co-operates with the power; then A B = 60 + 2 = 62; A C $= \frac{62}{2} = 31$; and C F = 60 — 31 = 29, therefore by the equality of moments,

$P \times 60 + 4 \times 29 = 180 \times 2$, ∴ $P = 4{\cdot}06$ lbs.

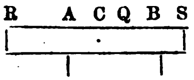
Ex. 2. Required the same as in the last example, when A F = 4 ft. *Ans.* 5·08*lbs.*

Ex. 3. If a pressure of 40 lbs. be applied at the extremity A, of the lever in Ex. 1., what weight will be balanced at the opposite extremity B? *Ans.* 10·1*cwts.*

Ex. 4. A lever of the first kind is 3 ft. long, and weighs 16 lbs.; a weight of 2 cwts. is placed at one extremity of the lever at the distance of 4 inches from the fulcrum. Where must a pressure of 30 lbs. be applied to balance the lever.

Ans. 22·4*inches from the fulcrum.*

Ex. 5. A beam R S, whose weight is 2 cwts. is supported by props at A and B, and a weight, Q, of 12 cwts. is placed at Q; it is required to determine the pressures upon the props, when R S = 40 ft., B S = 2 ft. Q B = 10 ft., and A B = 30 ft.



Let C be the centre of the beam, then C S $= \frac{1}{2}$ of 40 = 20, C B = 20 — 2 = 18,

And supposing the beam to turn upon B as a fulcrum, we have,

Pressure on A × 30 = 12 × 10 + 2 × 18 = 156.

∴ Pressure on A $= \frac{156}{30} = 5{\cdot}2$ cwts.

Now as the two props support the whole weight, or 14 cwt., we have, pressure on B = 14 — 5·2 = 8·8 cwts.

Ex. 6. Required the same as in the last example, when R S = 50 ft. *Ans.* 5·53 *and* 8·47*cwts.*

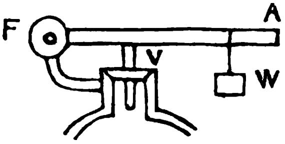
To graduate the Lever of a safety Valve.

31. The safety valve is intended to secure boilers from bursting by the elastic force of the steam. A F is a graduated lever of the second kind, turning upon F as a centre; V is the valve opening or closing, as the case may be, the communication of the steam in the boiler with the external air; the lever A F rests upon the pin of this valve, and a sliding weight W is suspended from the lever, thereby enabling the engine-man to place any amount of pressure upon the valve,—this pressure measures the elasticity of the steam when it begins to escape. In order to graduate the lever, we must first find the sliding weight W, placed at the extremity A, so as to sustain the greatest pressure of the steam; we then proceed to find the position of this weight to give any proposed pressure in the steam.

Ex. 1. The length A F of the lever is 20 inches, the distance between the pin of the valve V, and the fulcrum, F, is 2 inches, the weight of the valve is 4 lbs., the weight of the lever 5 lbs., and the section of the valve is 6 sq. inches; it is required to find the load W, so that when it is placed at the extremity of the lever, the steam may have a pressure of 50 lbs. per square inch.

Here total pressure of steam upon the valve $= 50 \times 6$ lbs.

$\therefore$ Effective pressure upon the lever $= 50 \times 6 - 4 = 296$ lbs.



Now the weight of the lever will act in its middle point; hence we have,

$$W \times 20 + 5 \times 10 = 296 \times 2; \ \therefore W = 27{\cdot}1 \text{ lbs.}$$

Ex. 2. At what distance from the fulcrum must the load in the last example be placed, so that the steam may have a pressure of 30 lbs. per sq. inch?

Effective pressure of the steam upon the lever $= 30 \times 6 - 4 = 176$ lbs.

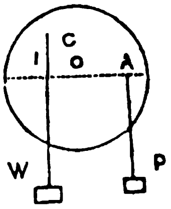
$\therefore$ 27·1 × W F + 5 × 10 = 176 × 2; $\therefore$ W F = 11·1 inches.

This distance, therefore, gives the position of the load when the pressure of the steam is 30 lbs.; proceeding in the same way the position may be determined for any other pressure.

Ex. 3. Required the positions of the weight, in Ex. 1, so that the steam may have the pressures of 40 and 20 lbs.

Ans. 15·5 *and* 6·7 *inches from the fulcrum.*

32 A board whose weight is neglected, turns upon the centre O, P and W are weights suspended from the points A and C: it is required to find the conditions of equilibrium.



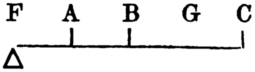
As every point in the cord C W has the same stretch or tension, we may drive a pin through the cord at I, in the horizontal line I O A, without altering the equilibrium, then I being now the point of suspension, the case proposed is reduced to that of a simple lever, and therefore the equation of equilibrium is W × O I = P × O A. Now it is no matter how the tensions of the cords C W and A P are produced: hence if we call the weight, or pressure, W, multiplied by the perpendicular O I, the moment of W, then it follows when equilibrium takes place, that the moments tending to turn the body in one direction, will be equal to the moments tending to turn the body in the contrary direction. This proposition expresses the principle of the equality of moments in its general form.

We may regard the perpendicular upon the direction of a force, as the virtual lever by which that force acts. In a *bent lever*, for example, we have merely to consider the perpendiculars upon the directions of the forces, as the virtual distances at which they act from the fulcrum.

Ex. 1. In a bent lever the perpendicular on the direction of P is 2 ft., and that upon the direction of W is 3 inches; required P when W is 160 lbs.

Here, P × 24 = 160 × 3; $\therefore$ P = 20 lbs.

as the force of gravity acting upon the various particles composing the body;—this point is called the centre of gravity of the body. From this definition it immediately follows; 1. That if the centre of gravity be supported, the body will will stand, and *vice versâ.* 2. That the centre of gravity of all symmetrical, or regular, bodies is in their centre of magnitude. 3. That if any body, acted on by the force of gravity, tend to turn a lever, we may regard the weight of the whole body to be collected in its centre of gravity.



Ex. 1. Let A, B, and C, be three bodies in the same right line, it is required to determine the position of their common centre of gravity G with respect to any assumed point F, when A = 1 lb., B = 2 lbs., C = 3 lbs., A F = 5, A B = 4, and B C = 6.

Let us suppose that an inflexible rod, without weight, passes through the bodies, and that the system turns upon F as a fulcrum; then as the whole mass may be regarded as acting in the point G, we have by the equality of moments,

$F\,G\,(1 + 2 + 3) = 1 \times 5 + 2 \times 9 + 3 \times 15$; $\therefore F\,G = 11\tfrac{1}{3}$.

Ex. 2. Required the same as in the last example, when A = 2 lbs., B = 4 lbs., C = 8 lbs., A F = 1, A B = 3, and B C = 10. *Ans.* *F G* = 9·2.

Ex. 3. Required the position of the centre of gravity of two weights P and W (See fig. page 53,) whose weights are 2 and 3 lbs. respectively, and distances apart 20 in. *Ans.* 12*in. from P.*

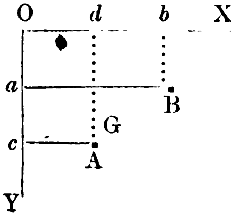
34. The centre of gravity of a triangle. Let A B C be a triangle having the vertex C; bisect the base A B in the point V, and the side B C in the point N; join the points C and V, A and N, V and N, and put G at the point where C V and A N intersect,—then G will be the centre of gravity of the triangle. From the description here given, the figure may be readily constructed.

The triangle will be balanced upon an edge lying along the line C V, because it will divide equally all lines drawn parallel to A B, hence the centre of gravity must lie in the line C V; for the same reason it will also lie in the line A N, and therefore it must be in the point G. Now V N = ½ A C, because the triangle V N B is simply the triangle A B C constructed on one-half the scale. Again, the triangle V G N is simply the triangle A G C constructed upon one-half the scale, (since they are equi-angular, and V N = ½ A C) therefore V G = ½ G C, or if V C be divided into 3 equal parts, V G will contain one of them. Hence, the distance of the centre of gravity of a triangle from the base, is ⅓ the elevation.

Similarly it may be shown that the centre of gravity of a pyramid is at ¼ the elevation.

35. The centre of gravity of any four-sided figure may be determined, by first drawing one diagonal and finding the centre of gravity of each of the triangles thus formed, then the centre of gravity of the whole figure must lie in the line joining these two points; in like manner, by drawing the other diagonal we determine another line in which the centre of gravity must lie; then the intersection of these two lines will give the centre of gravity of the whole figure.

36. A body will be more or less stable, according as it takes more or less *work*, to bring the line of direction, or a vertical line passing through the centre of gravity, without the base See Ex. 7, Art. 12. In general the lower the centre of gravity of a body is, the more stable it will be. Thus a boat is less liable to be overturned, when the passengers are sitting, than when they are standing. Hence it is, also, that a pyramid is the most stable form for a structure.



37. Let A, B, &c. be any number of bodies lying in the same horizontal plane; it is required to determine the position of the centre of gravity G, referred to two lines O X and O Y, perpendicular to each other, and called axes of coordinates.

Conceive the bodies to be connected with the axis O X, by the perpendicular rods A *d*, B *b*, &c. then the sum of the moments of the bodies, tending to turn round on the axis O X, will be the same as the moment of the whole mass, collected in the centre of gravity G. From this equality the *distance* of G from O X is obtained. In precisely the same way, we find the *distance* of G from O Y; and hence the *point* G becomes known. After the same manner the centre of gravity may be determined, when the bodies are referred to three coordinate planes.

Ex. 1. The weights of three bodies, A, B, and C, are 5, 6, and 8 lbs. respectively; and their distances from O X, 3, 4, and 7 ft., and from O Y, 2, 9, and 10 ft. respectively; required the position of the common centre of gravity.

Let X and Y be distances from O X and O Y respectively, then,

$(5 + 6 + 8)\,X = 5 \times 3 + 6 \times 4 + 8 \times 7; \therefore X =$ 5 ft.

$(5 + 6 + 8)\,Y = 5 \times 2 \times 6 \times 9 + 8 \times 10; \therefore Y$ $= 7{\cdot}5$ ft.

Ex. 2. Required the same as in the last example, when the weights of the bodies are 1, 2, and 3 cwts. *Ans.* 5¼ *and* 8¼*ft.*

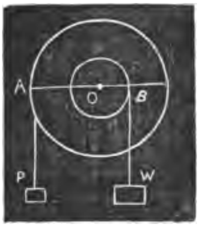
For further information on this subject see Moseley's Engineering, or Poisson's Mechanics.

Wheel and Axle.

38. This useful machine is only another form of the lever, where the power is made to act without intermission; in its most simple form, it consists of a large wheel, A, and a cylinder or axle, B, both of which turn upon the same axis, O. If the wheel, A, be turned round by a power, P, applied to it, the axle, B, will coil up the rope by which the weight W hangs. The lever by which P acts is evidently A O, and that by which W acts is B O; hence we have, when these weights, or pressures, balance each other, $P \times OA = W \times OB$.

If the wheel be displaced by a handle, then the machine is called a Windlass. Sometimes the handle is made to turn a series of wheels acting on each other by means of teeth, when the machine has this form it is called a Crane.

We proceed now to consider the equilibrium, on the principle of work.



While the wheel makes one revolution, the axle also makes one. In one turn P descends a space $= 2 \times OA \times 3{\cdot}1416$, and W ascends a space $= 2 \times OB \times 3{\cdot}1416$.

$\therefore$ Work of P in 1 revo. $= 2 \times OA \times 3{\cdot}1416 \times P$.

Work of W in 1 revo. $= 2 \times OB \times 3{\cdot}1416 \times W$.

Then as the work done (abstracted from friction) is equal to the work applied, we have

$$2 \times OA \times 3{\cdot}1416 \times P = 2 \times OB \times 3{\cdot}1416 \times W.$$

$$\therefore P \times OA = W \times OB,$$

which is the relation already established. Conversely assuming the equation of equilibrium, we may establish the principle of work.

Ex. 1. The handle of a windlass is 2 ft., the radius of the axle is 2½ inches, and the power applied 80 lbs.; required the weight which would be raised were there no friction.

Work of P in 1 revo. $= 80 \times 4 \times 3{\cdot}1416$.

" W $= W \times \frac{5}{12} \times 3{\cdot}1416$.

$\therefore W \times \frac{5}{12} \times 3{\cdot}1416 = 80 \times 4 \times 3{\cdot}1416$; and $W =$ 768 lbs.

Or thus, by the principle of the lever,

$80 \times 2 \times 12 = W \times 2\frac{1}{2}$; $W = 768$ lbs.

Ex. 2. What must be the diameter of the axle in the last example, so that the weight raised may be 500 lbs. *Ans.* ·64*ft.*

Ex. 3. If a man exert a pressure of 60 lbs. upon the handle of a windlass, what weight will he raise, when the length of the handle is 2 ft., and the radius of the axle 3 in. *Ans.* 480*lbs.*

Ex. 4. What is the advantage of pressure gained by the machine of the last example? *Ans.* 8.

Ex. 5. In example 3, if $\frac{1}{7}$ of the power is consumed by friction, what weight will then be raised? *Ans.* 411$\frac{3}{7}$*lbs.*

Ex 6. Required W in Example 1, when the thickness of the cord is 1 inch, supposing that $\frac{1}{7}$ of the work applied to be lost by friction and the rigidity of the cord.

Here the cord increases the radius of the axle by $\frac{1}{2}$ inch.

$\therefore$ Work of W in 1 revo. $= W \times \frac{6}{12} \times 3{\cdot}1416$.

Effective ,, P ,, $= \frac{6}{7} \times 80 \times 4 \times 3{\cdot}1416$.

$\therefore$ by equality, $W = \frac{80 \times 4 \times 12}{7} = 548$ lbs.

Ex. 7. In a crane the length of the handle is 1·4 ft., the number of teeth in the pinion 6, the number in the wheel 30, and the diameter of the axle $\frac{1}{3}$ ft.; if a pressure of 60 lbs. be applied to the handle, what must be the weight raised?

For every turn of the handle 6 teeth of the large wheel will be turned round, therefore as many times as the number of teeth in the pinion can be taken out of the number in the wheel, so many turns will the pinion make whilst the wheel makes one. Let the axle and wheel make one revolution, then the number made by the handle $= 30 \div 6 = 5$ revo.

$\therefore$ Space moved over by $W = \frac{1}{3} \times 3{\cdot}1416$ ft.

,, ,, $P = 2 \times 1{\cdot}4 \times 3{\cdot}1416 \times 5$.

$\therefore$ Work due to $W = \frac{1}{3} \times 3{\cdot}1416 \times W$.

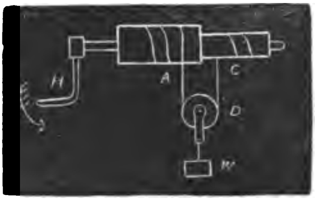
$\therefore \frac{1}{3} \times 3{\cdot}1416 \times W = 2 \times 1{\cdot}4 \times 3{\cdot}1416 \times 5 \times 60$;

$\therefore W = 2 \times 1{\cdot}4 \times 5 \times 60 \times 3 = 2520$ lbs.

Ex. 8. What must be the number of teeth in the wheel of the last example, so that a weight of 2 tons may be raised? *An* 53 *nearly.*

Compound Wheel and Axle.

39. In the common wheel and axle, there is a practical limit to the power of the machine; for we can only increase the power by increasing the size of the wheel, or by decreasing the radius of the axle. Now in the compound wheel and axle,



a given power may be made to raise any weight whatever. This machine consists of two axles A and C, cut upon the same block, round which a cord coils in opposite directions, this cord passes round the moveable pulley, D, which carries the weight W. Now when the handle is turned in the direction of the arrow, one of the cords is coiled upon the large axle A, while the other cord is uncoiled from the small axle C, so that the rate at which W ascends, depends upon the difference of the circumferences of the two axles; and, consequently, the power of the machine will also depend upon this difference. But this may bedecreased to any extent, without altering the length of the handle. This truly ingenious contrivance is due to the Chinese.

Ex. 1. Let the diameter of the axle A $= \cdot 7$ ft., the diameter of the axle C $= \cdot 5$ ft., the length of the handle $= 2$ ft., and $W = 400$ lbs. Required the power P.

When the handle makes one turn, the cord A will be drawn up a space equal to the circumference of the axle A, while the cord C will be let down a space equal to the circumference of the axle C; therefore the whole cord will be shortened a space equal to the difference of these circumferences, and because the cord is doubled, the weight W will be raised a space only equal to the half of this difference.

$\therefore$ Work done upon W in 1 revo. $= \dfrac{\cdot 7 \times 3\cdot 1416 - \cdot 5 \times 3\cdot 1416}{2} \times 400.$

$\therefore$ Work done upon P in 1 revo. $= 4 \times 3\cdot 1416 \times P.$

$$\therefore 4 \times 3\cdot 1416 \times P = \frac{\cdot 7 \times 3\cdot 1416 - \cdot 5 \times 3\cdot 1416}{2} \times 400,$$

dividing each side of the equality

$$4 \times P = \frac{\cdot 7 - \cdot 5}{2} \times 400; \quad \therefore P = 10 \text{ lbs.}$$

Ex. 2. In a compound wheel and axle, the handle is 20 in., the radius of the large part of the axle is 3 inches, that of the small part 2 inches; if 1 lb. pressure be applied to the handle, how many lbs. would be raised by the machine, friction being neglected?

Here, as the weight is supported by a double cord, and as one part acts with the power, we have by the equality of moments,

$$P \times 20 + \tfrac{1}{2} W \times 2 = \tfrac{1}{2} W \times 3, \text{ or}$$

$$1 \times 20 + W = \tfrac{3}{2} W, \ \therefore W = 40 \text{ lbs.}$$

Ex. 3. Solve the preceding question on the principle of *work.*

Ex. 4. Required the same as in Ex. 1, when the diameters of the axles are ·6 and ·4 ft. respectively, the length of the handle 1·5 ft., and the weight to be raised 600 lbs. *Ans.* 20*lbs.*

Ex. 5. What must be the radius of the large part of the axle in Ex. 2, so that the weight raised may be 160 lbs.

Ans. 2¼ *inches.*

40. From the reduced equation given in Ex. 1, we may readily derive a general equation of relation for the elements of this machine. For this purpose, put R for the length of the handle, D for the diameter of the large axle, and *d* for the diameter of the small one. Now ·7, in that equation, is the diameter of the large axle; ·5 the diameter of the small one; and 4 the length of the handle taken twice, hence we have,

$$2\,R \times P = \frac{D - d}{2} \times W$$

$$\therefore W = \frac{4\,R \times P}{D - d}$$

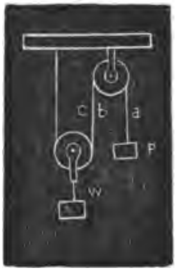
This expression clearly shows that other things being the same, the magnitude of W depends upon the smallness of the *difference* of the diameters of the axles.

The Pulley.

41. A pulley is a grooved wheel, turning on an axis, and placed in a block or case. A cord passes over the groove of

the wheel, in order to transmit the force applied, in any proposed direction. There is no advantage gained by a single fixed pulley; but when there are moveable pulleys, the weight raised will always be greater than the power applied; and then the advantage depends upon the number of cords by which the weight is suspended.

Ex. 1. In the annexed system of pulleys, if W = 2 lbs., required P, when equilibrium takes place.



Here, as W is suspended by two cords, *c* and *b*, each cord will carry 1 lb., but as the cord is supposed to have a free motion over the wheels, the portions *a*, *b*, and *c*, will have the same stretch or tension; therefore P must be 1 lb.

Let us now consider the equilibrium on the principle of the equality of work.

If W ascend 1 foot, the cords *c* and *b* will each be shortened 1 foot, and therefore P must descend 2 feet, hence we have,

Work of P = P × 2, and work of W = 2 × 1,

∴ P × 2 = 2 × 1, and P = 1 lb.

Ex. 2. Let there be two moveable pulleys, each weighing 3 lbs., then if P = 100 lbs., it is required to determine W, on the principle of work.

If P descend 4 feet, the first moveable pulley will ascend 2 feet, and the second 1 foot,

∴ Work done in raising W and the pulleys = W × 1 + 3 × 1 + 3 × 2 = W + 9.

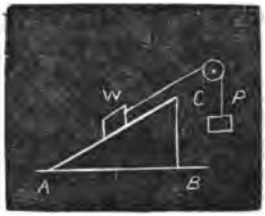
Work of P = 100 × 4, ∴ W + 9 = 100 × 4, and W = 391 lbs.

Ex. 3. Required the same as in the last example, when there are three moveable pulleys. *Ans.* 779 *lbs.*

In precisely the same manner the equation of equilibrium may be determined for any other system of pulleys.

Inclined plane.

42. To determine the pressure necessary to support a body on an inclined plane without friction. Let the weight, W, be drawn up the inclined plane A C, by means of the weight P, acting by a cord parallel to the plane; then whilst W is moved from A to C the counterpoise weight P will have descended a vertical space equal to A C, hence we have, by Art. 10,



Work in raising W = $W \times CB$.

Work due to the descent of P = $P \times AC$.

$\therefore P \times AC = W \times CB$, $\therefore P = \frac{CB}{AC} \times W$.

Ex. 1. The length of an inclined plane is 3 feet, the perpendicular height 2 feet, and the weight of the body 12 lbs., what pressure will be required to sustain the body on the plane, friction being neglected?

Work in raising W in opposition to gravity = 12×2.

Work of P = $P \times 3$; $\therefore P \times 3 = 12 \times 2$; and P = 8 lbs.

Ex. 2. Required the same as in the last example, when the length of the plane is 100 ft., the height 20 ft., and the weight 40 lbs. *Ans.* 8 *lbs.*

Ex. 3. The length of an inclined plane is 100 ft., the height 3 ft., the weight of the body 50 lbs., and the friction $\frac{1}{280}$ part of the weight, what pressure will be required to move the body up the plane?

In this case, as the inclination of the plane is small, the pressure upon it will be very nearly equal to the weight of the body.

$\therefore$ Pressure to overcome friction = $\frac{50}{280}$ lbs.

Work due to friction = $\frac{50}{280} \times 100 = 17{\cdot}8$.

Work due to gravity = 50×3.

Work of the pressure, P, applied to move the body = P × 100.

∴ P × 100 = 17·8 + 50 × 3, ∴ P = 1·67 lbs.

Ex. 4. Required the same as in the last example, when the length of the plane is 100 ft., the height 2 ft., and the weight of the body 56 lbs. *Ans.* 1·32*lbs.*

Ex. 5. What pressure will be necessary, in Ex. 3, to prevent the body from sliding down the plane?

Here the work due to friction will act with the work due to the pressure, P, applied,

∴ P × 100 = 50 × 3 — 17·8; ∴ P = 1·32 lbs

Ex. 6. The length of an inclined plane is 100 ft., the height ·2 ft., the weight of the body 90 lbs., and the coefficient of friction $\frac{1}{30}$; required the pressure necessary to move the body down the plane. *Ans.* 2·82*lbs.*

Here the work of gravity acts with the work of P.

Ex. 7. What must be the height of the plane in Ex. 3, so that the body may be upon the point of sliding?

In this case the work due to friction must be equal to the work due to gravity.

Work due to friction = 17·8.

Work due to gravity = 50 × C B.

∴ 50 × C B = 17·8; and C B = ·35 ft.

Ex. 8. What must be the height of the plane in Ex. 6, so that the body may be upon the point of sliding? *Ans.* 3⅓*ft.*

Obs. The inclination of the plane when the body is upon the point of sliding, is called the angle of friction, or "the limiting angle of resistance." See Moseley's Mechanics applied to the Arts.

The moveable Inclined Plane, or Wedge.

43. Let A C B be a wedge (See the last fig.) sliding along the horizontal plane A B by the action of a pressure, P, applied parallel to A B, and thereby elevating a weight W which is only free to move in a vertical direction. When the wedge begins to act, the resistance, W, rests upon the horizontal

plane A B; but when the wedge has moved over a space equal to its length, W will have been elevated a height equal to the thickness, C B of the wedge; then if A B = ·5 ft. B C = ·2 ft., and W = 8 lbs., we have, abstracted from friction,

Work applied = P × ·5, and W = 8 × ·2.

∴ P × ·5 = 8 × ·2, and P = 3·2 lbs.

This calculation shows that the advantage of the power depends upon the thinness of the back of the wedge. However, it is necessary to observe, that the astonishing power of the wedge, as usually employed, is chiefly resolvable into the force of impact, which will be considered in a future article.

Ex. 1. The length of a wedge = 1·5 ft., the thickness = ·2 ft., the pressure applied = 100 lbs.; required the weight that would be raised.

When the pressure applied has moved through the length of the wedge, we have,

Work applied = 100 × 1·5; Work done = W × ·2.

∴ W × ·2 = 100 × 1·5, and W = 750 lbs.

Ex. 2. Required the same as in the last example, when the length is 2 ft., the thickness ·4 ft., and the pressure applied 400 lbs. *Ans.* 2000 *lbs.*

The Screw.

44. In this mechanical power the pressure applied moves in a circle whose radius is the length of the lever, or arm of the screw, whilst the direction in which the work is done, is a right line.

Ex. 1. The lever of a simple screw is 2 feet, the thickness of the threads ·2 feet, if a pressure of 100 lbs. be applied to the lever, what pressure will be produced on the press board?

Here in one turn of the lever, the press board is moved over the distance between the threads of the screw.

Space moved over by P. in 1 revo. = 4 × 3·1416.
,, W ,, = ·2.

Work of P in 1 revo. = 4 × 3·1416 × 100.
,, W ,, = W × ·2.

∴ W × ·2 = 4 × 3·1416 × 100, and W = 6283 lbs.

Obs. The equation here obtained shows that the efficacy of the screw is increased by increasing the length of the lever, or by decreasing the thickness of the threads.

Ex. 2. Required the same as in the last example, when the thickness of the threads is ½ in., and the pressure applied to the lever is 60lbs. *Ans.* 18095 *lbs.*

Ex. 3. Required the advantage of the power in the last example. *Ans.* 301.

Ex. 4. The lever of a screw is 3 ft., and the thickness of the threads ¼ in.; required the pressure that must be exerted on the lever so as to produce a pressure of 10 tons upon the press board.

In 1 revo. of the lever the work done $= 10 \times 2240 \times \frac{1}{48} = 466$.

In 1 revo. of the lever the work applied $= P \times 6 \times 3{\cdot}1416$.

$\therefore P \times 6 \times 3{\cdot}1416 = 466$; and $P = 24{\cdot}7$ lbs.

Ex. 5. Required the same as in the last example, when the lever is 18 in., and the thickness of the threads of the screw 1 in. *Ans.* 198 *lbs.*

Ex. 6. What must be the thickness of the threads, in Ex. 1., so that a pressure of 5000 lbs. may be produced on the press board.

Work applied in 1 revo. $= 1256$.

Work done ,, $= 5000 \times$ thickness thread.

$\therefore 5000 \times$ thickness thread $= 1256$.

$\therefore$ Thickness thread $= \frac{1256}{5000} = {\cdot}25$ ft.

The Compound Screw.

45. This mechanical power consists of two screws, one of which screws within the other, so that whilst the large one is descending, the small one is relatively rising within the large one. In consequence of this compound motion, in one revolution of the lever, the press board only descends a distance equal to the difference of the thickness of the threads of the screws. Hence the advantage depends upon the smallness of this difference, and not upon the absolute size of the threads.

In this respect the machine is analogous to the compound wheel and axle.

Ex. 1. In a compound screw, the length of the lever is 1·5 ft., the distance between the threads of the large, or hollow screw, is $\frac{3}{4}$ in., and that of the small one $\frac{1}{2}$ in., if 30 lbs. pressure be applied to the lever, what will be the pressure on the press board?

In one revolution the large screw descends $\frac{3}{4}$ in., but at the same time, the small screw (by turning within the large one) ascends $\frac{1}{2}$ in.; therefore the press board must descend a space $= \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$ in. $= \frac{1}{48}$ ft.

$\therefore$ Work done in 1 revo. $= W \times \frac{1}{48}$.

Work applied in 1 revo. $= 30 \times 2 \times 1{\cdot}5 \times 3{\cdot}1416 =$ 28274.

$\therefore W \times \frac{1}{48} = 282{\cdot}74$; and $W = 13576$ lbs. nearly.

Ex. 2. Required the same as in the last example, when the threads is $\frac{7}{8}$ and $\frac{3}{4}$ in. respectively, and the pressure applied to the lever is 1 lb. *Ans.* 904

Ex. 3. What pressure must be applied to the lever in Ex. 1, so that a pressure of 4 tons may be given to the press board? *Ans.* 19·8 *lbs.*

Ex. 4. If the length of the lever is 2 ft., the thickness of the thread of the large screw 1 in., what must be the thickness of the small screw, so that 1 lb. applied to the lever may produce a pressure of $\frac{1}{4}$ ton?

Here, we have, by the principle of the equality of work,

$1 \times 4 \times 3{\cdot}1416 = 560\,(\frac{1}{12} - \text{thickness})$.

$\therefore$ Thickness $= {\cdot}06$ ft. $= {\cdot}72$ in.

The Endless Screw.

46. In this machine, the threads of a screw, cut upon a cylinder, are made to act upon the teeth of a cog wheel having an axle, round which a cord coils, as in the common windlass. By this combination a very slow motion may be given to the weight.

Ex. 1. In an endless screw the length of the handle is

1·4 ft. the number of teeth in the cog wheel is 30, and the radius of the axle is ·2 ft.; if 1 lb. pressure be applied to the handle, what weight will be raised, friction being neglected?

Here, as the screw is fixed upon the axis of the handle, one turn will cause one of the teeth of the wheel to be moved round. Hence the handle must make 30 turns, whilst the cog wheel and axle make one.

$\therefore$ Work applied in 1 revo. of the axle $= 1 \times 2{\cdot}8 \times 3{\cdot}1416 \times 30$

Work done " " $= W \times {\cdot}4 \times 3{\cdot}1416$.

$\therefore W = \frac{2{\cdot}8 \times 30}{{\cdot}4} = 210$ lbs.

Ex. 2. Required the weight, as in the last example, when the length of the handle is 1·5 ft., the number of teeth in the cog wheel 20, the radius of the axle ·3 ft., and the pressure applied 60 lbs. *Ans.* 6000 *lbs.*

Ex. 3. Required the pressure which must be applied to the handle, in the last example, so that a weight of 4500 lbs. may be raised. *Ans.* 45 *lbs.*

Ex. 4. How many teeth must the cog wheel, in Ex. 1, have in order to raise a weight of 280 lbs.? *Ans* 40.

Hydrostatic Press.

47. This powerful machine consists of two cylinders of different sizes, each of which has a piston, and pressure is transmitted from the small piston to the large one, by means of water. The pressure of the power is applied to the small piston, by a lever of the second kind; and the advantage of the machine depends upon the length of this lever, and the extent of surface of the large piston, as compared with that of the small one.

Ex. 1. In a hydrostatic press, the surfaces of the pistons are 2 and 400 sq. inches respectively, the lever is 20 inches long, and the piston rod is attached at 1 in. from the fulcrum; if a pressure of 112 lbs. be applied to the lever, what pressure will be produced upon the large piston?

Here the advantage of the lever is 20, and therefore 2240 lbs., or 1 ton, will be applied to the small piston; but owing to the transmission of this pressure through the water, every 2 inches of surface in the large piston will also undergo a pressure of 1 ton; hence it follows, that as many times as 2 inches can be taken out of 400 inches, so many times will the pressure of 1 ton be increased;

$$\therefore \text{Pressure on the large piston} = \frac{400}{2} \times 1 \text{ ton} = 200 \text{ tons}.$$

Or thus, on the principle of the equality of work.

Let the small piston descend $\frac{1}{2}$ in., or $\frac{1}{24}$ ft., then 1 cubic inch of water will be thrown into the large cylinder, and the piston will thus be raised $\frac{1}{400}$ part of an inch, or $\frac{1}{4800}$ of a foot;

$$\therefore \text{Work on the small piston} = 2240 \times \tfrac{1}{24}.$$
$$\text{,, large ,,} = W \times \tfrac{1}{4800}.$$
$$\therefore W \times \tfrac{1}{4800} = 2240 \times \tfrac{1}{24}; \text{ and } W = 200 \text{ tons}.$$

Ex. 2. Required the same as in the last example, when the surfaces of the pistons are $\frac{3}{4}$ and 600 inches respectively.

Ans. 800 *tons.*

Principle of Virtual Velocities.

48. If a very small motion be given to a machine in a state of equilibrium, then P multiplied by the velocity of P, estimated in the direction in which it acts, is equal to W multiplied by the velocity of W, estimated in the direction in which it acts;—this is the principle of virtual velocities. This proposition, is only another form of expressing the principle of the equality of work, developed in the preceding pages.

ACCUMULATED WORK.

Motion.

49. When a body moves uniformly, the space described is, obviously, equal to the units of time during which the body moves, multiplied by the space passed over in each unit of time; that is to say, the space is equal to the product of the

time by the velocity. If we, therefore, construct a rectangle, having the units in the base equal to the units in the velocity, and the units in the perpendicular equal to the units of time, then the units of surface in the rectangle, will give the units of space moved over by the body. In like manner, the space described by a body, whose motion is uniformly accelerated, may be represented by the area of a trapezoid, whose parallel sides contain respectively the units of velocity at the commencement and end of the motion, and the perpendicular between these sides the units in the time. This proposition, will readily be understood, by observing, that the mean length of the trapezoid will contain the same number of units as the velocity which the body has at the middle of the time; and this will be true however small the intervals of time may be taken. The best illustration of accelerated motion is afforded us in the case of falling bodies.

Force of Gravity.

50. When bodies fall freely near the surface of the earth, the force of the earth's attraction, being constant, communicates to them equal additions of velocity in equal intervals of time Thus at the end of 1 second, the velocity of the body is $32\frac{1}{6}$ ft.; at the end of 2 seconds, 2 times $32\frac{1}{6}$ ft.; at the end of 3 seconds, 3 times $32\frac{1}{6}$ ft.; and so on; or generally the velocity acquired by a falling body, is equal to the product of the time of the body's fall in seconds, by $32\frac{1}{6}$ ft. This is expressed by the equation, $v = t \times 32\frac{1}{6}$.

The space described by a body in one second will be $\frac{1}{2}$ of $32\frac{1}{6} = 16\frac{1}{12}$ ft.; because the velocity of the body in the middle of the time, will be the mean velocity with which it moves during that time. In like manner, the space described by the body in 4 seconds, will be 4 times $2 \times 32\frac{1}{6}$ ft.; because $4 \times 32\frac{1}{6}$ ft. is the velocity at the *end* of the time, and, therefore, $2 \times 32\frac{1}{6}$ ft. will be the mean velocity, or the velocity in the *middle* of the time. But 4 times $2 \times 32\frac{1}{6}$ ft. $= 4^2 \times 16\frac{1}{12}$ ft., that is the space described by a falling body, in 4 seconds, is equal to the square of the time multiplied by the space described in one second. In the same manner any other case may be established. The general relation of space and time is therefore expressed by,

$$s = t^2 \times 16\tfrac{1}{12}.$$

Ex. 1. What velocity will a falling body have at the end of 6 seconds?

Here the velocity = 6 times $32\frac{1}{6}$ ft. = 193 ft.

Ex. 2. What velocity will a body acquire in 3 seconds?

Ans. $96\frac{1}{2}$ *ft.*

Ex. 3. In what time will a body acquire a velocity of $128\frac{2}{3}$ ft.?

Here in every second of the body's fall, there is an addition of $32\frac{1}{6}$ ft. to its velocity, therefore as many times as $32\frac{1}{6}$ ft. can be taken out of $128\frac{2}{3}$ ft., so many seconds must the body fall, that is,

$$\text{Time} = 128\tfrac{2}{3} \div 32\tfrac{1}{6} = 4 \text{ sec.}$$

Ex. 4. In what time will a body acquire a velocity of 70 ft.?

Ans. 2·17 *sec.*

Ex. 5. Through what space will a body fall in 6 seconds?

Here, space = $6^2 \times 16\frac{1}{12}$ ft. = 579 ft.

Ex. 6. Through what space will a body fall in 3 seconds?

Ans. $144\frac{3}{4}$ *ft.*

Ex. 7. A stone was 4 seconds in falling from a tower; what height is it? *Ans.* $257\frac{1}{3}$ *ft.*

Ex. 8. If a body be projected downwards, with a velocity of 10 ft. per second, how far will it descend in 5 seconds?

It is evident that the body will retain its motion of projection, although acted upon by gravity. Now the space due to the projection = 5 times 10 ft., and this, added to the space due to gravity, will give,

$$\text{Space} = 5 \times 10 + 5^2 \times 16\tfrac{1}{12} = 452\tfrac{1}{12} \text{ ft.}$$

Ex. 9. Required the same as in the last example, when the velocity of projection is 5 ft, and the time of descent 2 seconds. *Ans.* $74\frac{1}{3}$ *ft.*

Ex. 10. In what time will a body fall 193 ft.

By the general equation, $t^2 \times 16\frac{1}{12} = 193$;

$$\therefore\ t^2 = \frac{193}{16\frac{1}{12}} = 12;\ \text{and } t = 3{\cdot}4 \text{ sec.}$$

Ex. 11. In what time will a body fall through 40 ft?

Ans. 1·57 *sec.*

Ex. 12. From what height must a body fall to acquire a velocity of $96\frac{1}{2}$ ft.

Here we must first find the time which the body will have to fall.

$$\text{Time} = 96\tfrac{1}{2} \div 32\tfrac{1}{6} = 3 \text{ sec}\ ;\ \text{then we have,}$$

$$\text{Space} = 3^2 \times 16\tfrac{1}{12} = 144\tfrac{3}{4} \text{ ft.}$$

Ex. 13. From what height would a body have to fall in order to acquire a velocity of 386 ft. *Ans.* 2316*ft.*

Ex. 14. The velocity of a body is 84 ft. From what height would it have to fall in order to attain this motion? *Ans.* 109*ft.*

$$\text{Time} = \frac{84}{32\frac{1}{6}} \text{ sec.; and space} = \left\{\frac{84}{32\frac{1}{6}}\right\}^2 \times 16\tfrac{1}{12} = \frac{84^2}{2 \times 32\frac{1}{6}}.$$

Observation. From this expression for the space we infer, that it is always equal to *the square of the velocity divided by* $2 \times 32\frac{1}{6}$.

Projectiles, &c.

51. We have a familiar instance of a body, in motion, acted upon by two forces, in a boat being rowed across a stream. While the stream carries the boat downwards, the rower moves it in a line perpendicular to the direction of the stream; now both of these motions exist at the same time, therefore the boat moves in a diagonal direction, that is to say, if we take one side of a parallelogram equal to the units of velocity in the stream, and the other side equal to the units of velocity of the boat crossing the stream, then the diagonal will give the direction and amount of the actual motion.

If a cannon ball be fired, in a horizontal direction, from the top of a high tower, the time it will take to reach the ground

Now the ball will not stop until all this work is destroyed, that is, until the work destroyed by friction is equal to the accumulated work.

Work destroyed by friction in moving the ball over 1 ft. $= \frac{20}{10} \times 1 = 2$.

$$\therefore \text{No. ft.} = \frac{1119{\cdot}17}{2} = 559{\cdot}58.$$

Ex. 5. Required the same as in the last example, when the velocity is 50 ft., and the weight 10 lbs. *Ans.* 388*ft.*

Ex. 6. A train weighs 50 tons, and has a velocity of 44 ft. per second when the steam is turned off; how far will the train move upon a level rail whose friction is 8 lbs. per ton.

$$U = \frac{44^2 \times 112000}{2 \times 32\frac{1}{6}} = 3370445.$$

When the train stops, the work of friction will be equal to the work accumulated in the train, hence we have,

Work in moving the train over every foot $= 50 \times 8 = 400$.

$$\therefore \text{No. ft.} = \frac{3370445}{400} = 8426.$$

Ex. 7. A train weighing 60 tons, has a velocity of 40 miles per hour when the steam is turned off, how far will it ascend an incline of 1 in 100, taking friction at 8 lbs. per ton.

Ans. ·74 *miles.*

In this example the work of gravity must be added to the work of friction to obtain the total work of the resistance.

Ex. 8. Required the same as in the last example, when the velocity is 20 miles, and the incline has a rise of 1 in 30.

Ans. 362·4*ft.*

Ex. 9. Two balls each weighing 50 lbs., are placed at the extremities of a horizontal arm, which gives motion to a screw driving a punch, as in the common stamping machine. The velocity given to the balls is 10 ft. per second. It is required to find the mean resistance opposed to the punch, when it is just driven through an iron plate $\frac{1}{2}$ in. in diameter.

$$\text{Here, } U = \frac{10^2 \times 100}{2 \times 32\frac{1}{6}} = 155{\cdot}44.$$

Now if the thickness of the plate were 1 foot, the resistance would obviously be 155·44 lbs.; but the thickness is $\frac{1}{96}$ of a foot

∴ Mean resistance = 96 times 155·44 = 14922 lbs.

Ex. 10. A ball weighing 20 lbs., is fired from the mouth of a cannon 10 ft. long, with the velocity of 500 ft. per second; it is required to find the mean pressure of the elastic vapour upon the ball.

In this case, U = 77720.

Now the pressure of 1 lb. upon the ball would produce 10 units of work.

$$\therefore \text{ Mean pressure} = \frac{77720}{10} = 7772 \text{ lbs.}$$

Ex. 11. Required the same as in the last example, when the ball is 4 lbs., and the velocity 200 ft. *Ans.* 248·7 *lbs.*

Ex. 12. A carriage of 1 ton moves on a level rail with the speed of 8 ft. per second, through what space must the carriage move to have a velocity of 2 ft., supposing friction to be 6 lbs. per ton?

$$\text{Here the work lost} = \frac{8^2 \times 2240}{64\frac{1}{3}} - \frac{2^2 \times 2240}{64\frac{1}{3}} = 2089.$$

But this work has been taken up by friction.

∴ Space × 6 = 2089; and space = 348.

Ex. 13. If the carriage in the last example move over 400 ft. before it comes to a state of rest; what is the resistance of friction per ton?

Work accumulated in the carriage = 2228.

Work of friction = friction × 400.

∴ Friction × 400 = 2228; and friction = 5·5 lbs.

Ex. 14. Two weights W and *w*, weighing 5 and 3 lbs. respectively, are connected by a cord that goes over a fixed pulley, as in Attwood's machine; through what space must W descend to acquire a velocity of 10 ft.?

As the velocity with which *w* ascends is the same as the velocity with which W descends, we have,

$$\text{Work in W} = \frac{10^2 \times 5}{64\frac{1}{3}}\text{; Work in } w = \frac{10^2 \times 3}{64\frac{1}{3}}$$

$$\therefore \text{ Total accumulated work} = 12{\cdot}4.$$

$$\text{Work of gravity on W} = 5 \times \text{space.}$$

$$\text{,,} \quad \text{,,} \quad w = 3 \times \text{space.}$$

But this work performed upon w has been yielded by the work of W.

$$\therefore \text{ Work remaining in the bodies} = 2 \times \text{space.}$$

Now this work must be equal to the accumulated work,

$$\therefore 2 \times \text{space} = 12{\cdot}4\text{, and space} = 6{\cdot}2 \text{ ft.}$$

Work in a rotating body.—Centre of gyration.

58. *Problem.* Two balls A and B are connected by a rod which is made to revolve upon a centre, the weight of A is 3 lbs., and that of B is 4 lbs., the distance of A from the axis is 8 ft., and that of B is 5 ft.; if a point in the rod, at 1 foot from the axis, has a velocity of 10 feet per second; it is required to determine the work in the balls, and the point in the rod where we may suppose the weight of the two balls collected, so that the work may not be altered.

$$\text{Velocity A} = 8 \times 10\text{; Velocity B} = 5 \times 10\text{;}$$

$$\text{Work in A} = \frac{8^2 \times 10^2 \times 3}{64\frac{1}{3}}\text{; Work in B} = \frac{5^2 \times 10^2 \times 4}{64\frac{1}{3}}.$$

$$\therefore \text{ Total work} = 453{\cdot}8.$$

Let d be the distance from the axis, then

$$\text{Work of the two balls collected} = \frac{(10\,d)^2 \times 7}{64\frac{1}{3}}$$

Now this work must be equal to the work before given, hence by equality we readily find, $d = 6{\cdot}4$ ft.

This point is called the *centre of gyration.* When the centre of gyration in a rotating body is known, we can then very readily find the accumulated work by the method already given. But to find this centre generally, requires the aid of the integral calculus, which would be foreign to the objects of this work to introduce. The distance of the centre of gyration from the axis, in a few of the most useful cases is as follows:

in a circular wheel of uniform thickness, it is equal to the radius of the wheel × $\sqrt{\frac{1}{2}}$; in a rod revolving about its extremity, it is equal to the length of the rod × $\sqrt{\frac{1}{3}}$, and when it revolves about its centre, it is equal to the length × $\sqrt{\frac{1}{12}}$; and in a plane ring, like the rim of a fly wheel, it is equal to the square root of one half of the sum of the squares of the radii forming the ring.

Ex. 1. The weight of a fly wheel is 10 cwts. This weight is collected in a point at the distance of 5 ft. from the axis, or in other words, the centre of gyration is 5 ft. from the axis. The wheel makes 20 revolutions per min., the diameter of the axis is 2 inches, and the friction upon it $\frac{1}{7}$ of the whole weight. How many revolutions will the wheel make before it stops?

Ans. 22·7.

Velocity of the wt. per sec. $= 10 \times 3{\cdot}1416 \times 20 \div 60 = 10{\cdot}47$.

$$\text{Work in the wheel} = \frac{10{\cdot}47^2 \times 1120}{64\frac{1}{3}}.$$

$$\text{Circumference axis} = \tfrac{2}{12} \times 3{\cdot}1416 = {\cdot}5236.$$

$$\text{Work destroyed by friction in 1 revo.} = \frac{1120}{7} \times {\cdot}5236.$$

Then, as many times as this work can be taken out of the work in the wheel, so many revolutions will it make.

Ex. 2. Required the same as in the last example, when the wheel makes 10 revolutions per min., and the diameter of the axis is 3 inches. *Ans.* 3·8.

Ex. 3. The weight of a fly wheel is 4 cwts., the distance of the centre of gyration from the axis is 4 ft., and the number of revolutions made per min. is 40; it is required to find the number of strokes which the fly wheel will give to a forge hammer, whose weight is 200 lbs., and lift 2 ft., friction being neglected.

Velocity of the weight per sec. = 16·755 ft.

$$\text{Work in the wheel} = \frac{16{\cdot}755^2 \times 448}{64\frac{1}{3}} = 1954{\cdot}9.$$

$$\text{Work in 1 lift of the hammer} = 200 \times 2 = 400.$$

$$\therefore \text{No. lifts} = \frac{1954{\cdot}9}{400} = 4{\cdot}8.$$

Ex. 4. Required the same as in the last example, when the circumference of the axis is 6 inches, and the friction upon it $\frac{1}{8}$ of the weight. *Ans.* 4·5.

Ex. 5. The diameter of a grindstone is 4 ft., and its weight 300 lbs. The circumference is made to revolve with the velocity of 5 ft. per second. The circumference of the axis is 6 inches, and the friction upon it $\frac{1}{8}$ of the weight. It is required to find the number of revolutions, which the stone will make, when left to itself.

$$\text{Centre of gyration from the axis} = 2\sqrt{\tfrac{1}{2}}.$$

$$\therefore \text{Velocity of the weight per sec.} = \frac{5}{2} \times 2\sqrt{\tfrac{1}{2}}.$$

$$\text{The square of this velocity} = \frac{25}{2}.$$

$$\therefore \text{Work in the stone} = \frac{25 \times 300}{2 \times 64\frac{1}{3}} = 58{\cdot}3.$$

$$\text{Work destroyed in 1 revo. by friction} = \frac{6}{12} \times \frac{300}{8} = 18{\cdot}7.$$

$$\therefore \text{No. revo.} = \frac{58{\cdot}3}{18{\cdot}7} = 3{\cdot}1.$$

Ex. 6. Required the same as in the last example, when the diameter of the stone is 3 ft., the diameter of the axis 1 in., and the stone makes one revolution per second. *Ans.* 1·6.

Vis Viva, or Living Force.

54. Let the weights of two bodies, A and B, be 4 and 7 lbs. respectively, and their velocities 5 and 9 ft. per second; required the accumulated work of the bodies, in terms of their vis viva.

$$\text{Work in A} = \frac{5^2 \times 4}{2 \times 32\frac{1}{6}};\ \text{work in B} = \frac{9^2 \times 7}{2 \times 32\frac{1}{6}}$$

$$\therefore \text{Accumulated work in A and B} = \frac{1}{2} \times \frac{5^2 \times 4 + 9^2 \times 7}{32\frac{1}{6}}$$

Now the latter part of the expression, in the right hand side of this equality, is called the vis viva of the bodies A and B.

$\therefore$ Accumulated work $= \frac{1}{2}$ the vis viva.

55. In the preceding problem, what must be the weight of another body W, having a velocity of 8 ft. per second, so that its accumulated work may be the same as that of the two bodies.

Here the work in W $= \dfrac{8^2 \times W}{2 \times 32\frac{1}{6}}$; but this will be equal to the work in the bodies just given; therefore, by an easy reduction, we have,

$$8^2 \times W = 5^2 \times 4 + 9^2 \times 7, \text{ and } W = 10{\cdot}4 \text{ lbs.}$$

Observation. Now if the bodies A, B, and W are parts of the same machine, and if the velocity of A be increased, the velocity of B and W will obviously be increased in the same ratio: hence it follows, from the expression, that W will not be altered by any change in the velocity of the machine.

Maximum velocity of the Piston of an Engine.

56. This will obviously take place when the pressure of the steam in the cylinder is equal to the total pressure of the resistances upon the piston; for, so long as the pressure of the steam is greater than the resistances, the motion of the piston must be accelerated. Then the work remaining in the piston, at this point of the stroke, will be equal to the work accumulated in the machine. From this equality the velocity may be determined.

Ex. 1. Required the maximum velocity of the piston, in Ex. 4, Art. 25, supposing the weight of the mass moved, calculated to have the same motion as the piston, to be 30000 lbs

In this case, we have found the total resistances to be 15 lbs. per inch nearly, then by Marriotte's law, the position of the piston, for the steam to have this pressure, must be 2 ft. from the bottom of the cylinder; hence we find the total work of the steam up to this point, to be 62352. But the work of the resistances up to this point $= 15 \times 1\frac{3}{4} \times 1440 = 37800$,

$\therefore$ Work remaining $= 62352 - 37800 = 24552$.

Now this excess of work is accumulated in the mass in motion.

$$\therefore \frac{V^2 \times 30000}{64\frac{1}{3}} = 24552, \text{ and } V = 7{\cdot}2 \text{ ft.}$$

Similarly the velocity may be found for any point of the stroke.

The weight of the mass moved, in any engine, referred to the piston, may be determined by Art. 55.

See Report of Experiments on the Steam Engine given at the tenth meeting of the British Association.

Velocity acquired by a body descending an inclined plane.

57. If a body freely descend an inclined plane by the force of gravity alone, the work accumulated in the body, by Art. 10, will be simply that which is due to the perpendicular elevation, without any regard to the angle, or curve of the plane. Hence the velocity of the body, will be that which it would acquire by falling freely through the perpendicular height.

Ex. 1. What velocity would a body acquire in descending a plane whose elevation is 144¾ ft.

Here the time which the body would take to descend this height by Art. 50, is 3 seconds, therefore the velocity acquired $= 3 \times 32\frac{1}{6} = 96\frac{1}{2}$ ft.

Ex. 2. What velocity will a body acquire in descending a plane whose height is 8 ft.? *Ans.* 22·6 *ft.*

When the friction of the plane is taken into account, it will be most convenient to employ the usual expression for accumulated work.

Ex. 3. A train of 10 tons descends an incline of 200 ft., and having a total rise of 3 ft.; what will be the velocity acquired by the train, supposing the friction to be 8 lbs. per ton?

Work in the train $= 10 \times 2240 \times 3 - 10 \times 8 \times 200$

$$\therefore \frac{V^2 \times 10 \times 2240}{64\frac{1}{3}} = 10 \times 2240 \times 3 - 10 \times 8 \times 200.$$

Hence we find, $V = 12{\cdot}1$ ft.

To determine the Friction upon an Axis by experiment.

58. *Example.* The wheel in Ex. 1, Art. 53, was observed to make 28 revolutions before it came to a state of rest; what part of the weight of the wheel is the friction upon the axis?

Let F be put for this proportional part, then the work destroyed by friction $= F \times 1120 \times {\cdot}5236 \times 28$.

But this work must be equal to the work accumulated in the wheel, hence from the resulting equation we find $F = {\cdot}11$.

Resistance of Fluids.

59. It is here proposed to establish the law of resistance assumed in Art. 14. The resistance of a fluid to the motion of a body is occasioned by the force necessary to displace that fluid. Now the fluid displaced must have the same motion given to it as that of the moving body; hence the work destroyed by the fluid will be equal to the accumulated work in the fluid. Let, for example, the front of the body, presented to the fluid, contain 2 sq. ft., the weight of a cubic foot of the fluid 62 lbs., and the velocity of the body 9 ft. per second,—then the weight of the fluid displaced every second will be $9 \times 2 \times 62$ lbs., but this weight has a velocity of 9 ft. given to it, therefore the work expended in the displacement $= \dfrac{9^2 \times 9 \times 2 \times 62}{64\frac{1}{3}}$. Now this work has been destroyed while the body has moved through the space of 9 ft.; if we call R the resistance of the fluid in lbs., this work will also be represented by $R \times 9$, therefore by equality

$$R \times 9 = \frac{9^2 \times 9 \times 2 \times 62}{64\frac{1}{3}}, \text{ and } R = \frac{9^2 \times 2 \times 62}{64\frac{1}{3}}.$$

Where we observe, that the resistance increases with the square of the velocity, as well as with the extent of surface presented to the fluid. In extreme velocities this law does not hold strictly true. It appears also from certain recent railway experiments, that the resistance of the atmosphere, to the motion of the train, depends chiefly upon the length of the train, and not so much upon the extent of the frontage of the carriages. The resistance of the atmosphere, given in Art. 14, must therefore be considered as only true for a certain length of the train.

Efflux of Fluids.

60. If an aperture be made in the side of a vessel, kept full of water, then the accumulated work of the fluid, as it is being discharged, will just be equal to the work which gravity would perform upon it, while descending a space equal to the perpendicular depth of the orifice. Hence it follows, that the velocity of discharge is equal to the velocity which a body would acquire in falling freely through a space equal to the perpendicular depth of the orifice.

Ex 1. How many cubic ft. of water will a pipe, 1 inch in the section, discharge per min., from a vessel kept constantly full, the depth of the discharge pipe being 9 ft.?

Let V be put for the velocity of discharge per sec., and w the weight of a fluid particle in lbs., then,

$$\text{Accumulated work in this particle} = \frac{V^2 \times w}{64\frac{1}{3}}$$

$$\text{Work of the particle in descending 9 ft.} = 9 \times w.$$

$$\therefore \frac{V^2 \times w}{64\frac{1}{3}} = 9 \times w, \text{ and } V = \sqrt{64\frac{1}{3} \times 9} = 24 \text{ ft.}$$

$$\therefore \text{Discharge per min.} = \tfrac{1}{144} \times 24 \times 60 = 10 \text{ c. ft.}$$

Ex. 2. If the section of the cistern, in the last example, be 5 ft., and a pressure of 3225 lbs. be applied to a piston fitting the cistern and in contact with the water; what will be the velocity of the discharge?

Here it is evident, that the effect of the pressure of the piston upon the efflux will be equivalent to an additional column of fluid producing the given pressure of the piston.

$$\text{Additional col. of fluid} = \frac{3225}{5 \times 62{\cdot}5} = 10 \text{ ft.}$$

$$\text{Total col. of fluid} = 9 + 10 = 19 \text{ ft.}$$

Then, proceeding as in the last example, we have,

$$V = \sqrt{64\tfrac{1}{3} \times 19} = 35 \text{ ft. nearly.}$$

Observation. In the preceding calculations no account has been taken of the vena contracta, or the contraction of the stream at the discharge.

Work performed by a jet of Steam.

61. When steam issues from a nozzle, its elasticity is nearly the same as that of the surrounding atmosphere; and the volume of the steam at the nozzle will therefore be 1711 times that of the water from which it is raised. The motion of the particles of the steam, at the nozzle, is due to the work of the expansion of the steam.

Ex. 1. Steam is discharged from a nozzle, whose area is 2 inches, with the velocity of 500 ft. per second; how many H. P. would a wheel perform which takes up all the work in the steam?

$$\text{Vol. steam discharged per sec.} = \tfrac{2}{144} \times 500.$$

$$\text{Wt. ,, ,, ,,} = \frac{2 \times 500 \times 62{\cdot}5}{144 \times 1711} = {\cdot}2536 \text{ lbs.}$$

$$\text{Accumulated work per sec.} = \frac{500^2 \times {\cdot}2536}{64\frac{1}{3}} = 985.$$

$$\therefore \text{H. P.} = \frac{985 \times 60}{33000} = 1{\cdot}7.$$

Ex. 2. If 9 cubic feet of water be evaporated per hour, when the area of the nozzle is 1 inch; how many H. P. will the wheel have?

$$\text{Vol. steam discharged per sec.} = \frac{1711 \times 9}{60 \times 60}$$

$$\therefore \text{Velocity steam per sec.} = \frac{1711 \times 9 \times 144}{3600} = 68{\cdot}4 \times 9$$

$$\text{Wt. steam discharged per sec.} = \frac{9 \times 62{\cdot}5}{3600} = {\cdot}017 \times 9.$$

$\therefore$ Accumulated work per sec. $= \dfrac{(68{\cdot}4 \times 9)^2 \times {\cdot}017 \times 9}{64\frac{1}{3}}$
$= 1{\cdot}23 \times 9^3$.

From this expression it appears, that the work performed by steam, in this manner, increases with *the cube of the water evaporated;* this remarkable result is due to Professor Moseley.

$$\text{H. P.} = \frac{1{\cdot}23 \times 9^3 \times 60}{33000} = 1{\cdot}6.$$

Observation. In these calculations the coefficient of efflux has been neglected.

Work of the Fire Engine.

62. In this engine, water is thrown from a pipe with a velocity necessary to reach a certain distance; in this water, therefore, there is accumulated work, the amount of which may be determined by the usual expression.

Ex. 1. The section of a pipe of a fire engine is ¾ in., and the velocity of the water discharged 50 ft. per second; required the work which must be applied to the engine per minute.

Here wt. water discharged per min. $= \dfrac{3}{4 \times 144} \times 50 \times 60 \times 62{\cdot}5$.

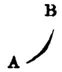
$\therefore$ Accumulated work per min. $= \dfrac{50^2 \times 3 \times 50 \times 60 \times 62{\cdot}5}{4 \times 144 \times 64\frac{1}{3}}$
$= 37949$.

Ex. 2. How many men will it take to work the engine in the last example, allowing that a man can perform 2600 effective units of work per min.?

$$\text{No. men} = \frac{37949}{2600} = 14{\cdot}5.$$

Ex. 3. Required the same as in Ex. 1, when the section of the pipe is 1 in., and the water is projected with a velocity of 120 ft. *Ans.* 699437.

Poncelet's Water Wheel.



63. In the common undershot water wheel, the paddles are flat, whereas in Poncelet's wheel they have a curved shape, A B; so that the direction of the curve at A, where the water first meets the paddle, is the same as the direction of the stream. By this ingenious contrivance, the water rolls up the curved incline A B, without meeting with any sudden obstruction calculated to occasion a loss of work. The channel has a depression at the point where the water falls from the paddles. Let V be the velocity of the stream, and v that of the wheel, then since the point A of the paddle is moving away from the stream, the water will flow upon the paddle with the velocity $V - v$, and will continue to run up the curved incline until it has lost its motion, it will then descend, acquiring in its descent the same velocity as that which it had in its ascent, but in a contrary direction. If the wheel were not moving, $V - v$ would be exactly the velocity of discharge from right to left, but the paddle is moving the water from left to right with the velocity v, therefore the absolute velocity of the water upon leaving the paddle will be $V - v - v = V - 2v$. Now all the work will have been taken out of the water, when its motion upon leaving the paddle is nothing, that is, when $V - 2v = 0$, or $V = 2v$. In this case, the water having lost all its motion, will simply drop from the paddle, and the work done upon the wheel, will be equal to the work accumulated in the water of the stream. Moreover it appears, that this maximum condition is fulfilled when the velocity of the stream is double that of the wheel. However the distinguished inventor states, that, in practice, the velocity of the water, in order to produce its maximum effect, ought to be about 2½ times that of the wheel, and that then the modulus of the wheel is about ·7. This wheel, other things being the same, will perform about twice the work of the common undershot wheel.

When there is motion in the water after leaving the wheel, we must calculate the work accumulated in that water, and subtract it from the whole work originally in it, in order to obtain the work done upon the wheel. Proceeding in this way, after a little reduction, we find $U = \frac{2\,W}{g}(V - v)\,v$.

Ex. 1. Ninety cubic feet of water flow upon the paddles of Poncelet's wheel per second, with the velocity of 6 ft., when the wheel moves at the rate of 3 ft. per second; required the H. P. of the wheel.

This is obviously a case of maximum work.

Weight water flowing per sec. $= 90 \times 62{\cdot}5 = 5625$.

$$\therefore \text{Work per sec.} = \frac{6^2 \times 5625}{64\frac{1}{3}} = 3147.$$

Hence we find the H. P. $= 5{\cdot}7$.

Ex. 2. Required the same as in the last example, when 70 cubic feet of water flow upon the paddles. *Ans.* 4·4

Ex. 3. Required the same as in the preceding examples, when the quantity of water is 80 c. ft., the velocity of the stream 3 ft. and that of the wheel 1 ft. *Ans.* 1·13.

Here the velocity with which the water leaves the paddles $= 3 - 2 \times 1 = 1$ ft.

$$\therefore \text{Work remaining in the water} = \frac{1^2 \times 80 \times 62{\cdot}5}{64\frac{1}{3}}.$$

$$\text{Work in the water at first} = \frac{3^2 \times 80 \times 62{\cdot}5}{64\frac{1}{3}}.$$

Taking the difference of these expressions we find the work done upon the wheel per second to be 621·7. This result might have been more easily calculated from the general expression for U above given.

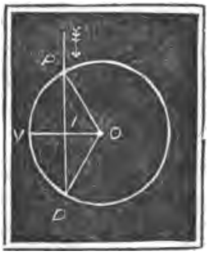
$$\therefore \text{H. P.} = \frac{621{\cdot}7 \times 60}{33000} = 1{\cdot}13.$$

Ex. 4. Required the same as in the last example, when the water flowing per second is 60 cubic feet, the velocity of the stream 9 ft., and that of the wheel 4 ft. *Ans.* 8·47.

Observation. In these questions no account is taken of friction and other resistances.

THE FLY WHEEL.

To find the position of the crank corresponding to its maximum and minimum velocity in a single acting engine.



64. Let O P and O D be the required positions of the crank, and let us suppose P to be the constant pressure of the connecting rod acting always in a vertical line. Put Q for the *constant* resistance, acting at one foot from the axis of the fly-wheel, equivalent to the work of the engine. The motion will be accelerated from P to D. This acceleration will commence when the moving pressure is equal to the resisting pressure, and it will cease under the same condition. The former will correspond to the position of minimum, the latter to that of the maximum velocity. Hence at these two points the moment of P must be equal to the moment of Q, and the point D will be as much below the horizontal line O V as the point P is above it.

$$\therefore P \times OI = Q \times 1.$$

Again, we have by the equality of work, putting r for O P,

$$\text{Work of P in 1 revo.} = 2\,r\,P.$$

$$\text{,,} \quad Q \quad \text{,,} \quad = Q \times 2 \times 3{\cdot}1416.$$

$$\therefore 2\,r\,P = Q \times 2 \times 3{\cdot}1416 \ldots (1).$$

Dividing the former equation by the latter, we have,

$$\frac{OI}{r} = \frac{1}{3{\cdot}1416} = {\cdot}3183.$$

Now this is the cosine of the angle P O V, hence from the tables, we find P O V = 71° 27′.

To find the dimensions of the Fly Wheel.

65. Let d and p, be the maximum and minimum velocities of the wheel at the distance of 1 from the axis; W the weight of the wheel, and k the distance of the centre of gyration from the axis.

Work of P from P to D $= \mathrm{P} \times \mathrm{PD} = 2\,r\,\mathrm{P}\sin.\ 71^\circ\ 27' = 2\,r\,\mathrm{P} \times {\cdot}948$.

$$\text{Work of Q from P to D} = \frac{\mathrm{Q} \times 2 \times 3{\cdot}1416 \times 142^\circ\ 54'}{360^\circ}.$$

$$= 2\,r\,\mathrm{P} \times {\cdot}3968,$$

By substituting the value given in equation (1) Art. 64, and reducing.

Now the difference of these will give us the work that goes to increase the speed of the wheel between the points P and D, that is

Work going into the wheel between P and D =

$$2\,r\,\mathrm{P} \times {\cdot}948 - 2\,r\,\mathrm{P} \times {\cdot}3968 = r\,\mathrm{P} \times 1{\cdot}1022.$$

$$\text{Accumulated work at P} = \frac{p^2\,k^2\,\mathrm{W}}{2\,g}$$

$$\text{,, at D} = \frac{d^2\,k^2\,\mathrm{W}}{2\,g}.$$

$$\therefore \text{Accumulated work gained from P to D} = \frac{k^2\,\mathrm{W}}{2\,g}(d^2 - p^2).$$

But this must be equal to the work before found.

$$\therefore \frac{k^2\,\mathrm{W}}{2\,g}(d^2 - p^2) = r\,\mathrm{P} \times 1{\cdot}1022 \ldots . (1)$$

Let V be the mean velocity of the wheel at 1 from the axis, and let the velocities d and p, at each of the extremes differ from this mean by an n$^{\text{th}}$ part; then

$$d = \mathrm{V} + \frac{\mathrm{V}}{n},\ p = \mathrm{V} - \frac{\mathrm{V}}{n}, \text{ and}$$

$$\therefore d^2 - p^2 = \frac{4\,\mathrm{V}^2}{n} \ldots (2).$$

Let also U be the work of the engine, and N the number of strokes performed per minute; then

$$\mathrm{V} = 2 \times 3{\cdot}1416 \times \frac{\mathrm{N}}{60} = {\cdot}10472 \times \mathrm{N} \ldots (3).$$

$$\mathrm{U} = 2\,r\,\mathrm{P}\,\mathrm{N},\ \therefore r\,\mathrm{P} = \frac{\mathrm{U}}{2\,\mathrm{N}} \ldots (4).$$

Substituting the values given in the equations (2), (3), and

(4), in equation (1), and reducing the given quantities we finally obtain,

$$W = \frac{n\,U}{k^2\,N^3} \times 808{\cdot}2 \;\ldots\; (5).$$

After the same manner we may derive the equation, expressing the relation of the elements, for the double acting engine. This result, obtained without the aid of the differential calculus, is substantially the same as that given in Moseley's Engineering, page 385. For further details on this interesting subject, see the work just referred to, or Hann's Treatise on the Steam Engine.

Ex. 1. The H. P. of a single acting engine are 30, the number of strokes performed by the piston per minute is 20, the distance of the centre of gyration of the fly-wheel from the axis 10 feet; it is required to find the weight of the wheel, so that the velocity of each revolution may not vary by more than $\frac{1}{5}$ from the mean.

Here $U = 30 \times 33000$; $k = 10$; $N = 20$; and $n = 5$.

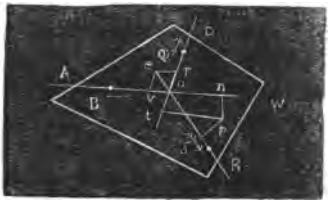
Substituting these values in eq. (5) we obtain, $W = 2{\cdot}23$ tons.

Ex. 2. Required the same as in the last example, when $U = 330000$; $k = 10$; $N = 20$; and $n = 15$.

Ans. 2·2 *tons.*

EQUILIBRIUM OF PRESSURES, &c.

Parallelogram of Forces.



66. Let the board A W R be placed upon three balls rolling freely upon a horizontal table. Take any three points, B, Q, and S, in the surface of the board, and let the cords A B, D Q, and R S be attached to the points. These cords pass over pulleys, and have different weights

hanging at their extremities. Under this arrangement the board, acted upon by three forces, will roll upon the table until it comes to a position where the three forces destroy each other. When this position has been attained, the following important relation will be observed, between the direction and magnitude of the forces.

1. The direction of the forces will intersect, or meet, in the same point *o*.

2. From a scale of equal parts take off *o r* equal to the units of pounds in the weight, or force, drawing by the cord Q D, and in like manner from the same scale of equal parts take off *o v* equal to the units of pounds in the weight, or force, drawing by the cord A B, then if upon the two lines *o r* and *o v* we construct the parallelogram *o r e v*, the diagonal *o e* will be the *direction* of the third force acting by the cord R S, and the units of length in the diagonal *o e* will give the units of pounds in the weight, or force acting by this cord. This is called the principle of the parallelogram of forces. The forces represented by the sides *o r* and *o v* are called the component forces, and the force just equivalent to these two and represented by *o e*, is called the resultant.

3. Instead of three forces being applied to the board, let there be any number. Let P be any point taken in the board, and from this point let fall the perpendiculars P *n*, P *s*, P *t*, &c. on the directions of the forces, or if necessary, on the directions of the forces produced; then the units of length in any one of these perpendiculars, multiplied by the units of pounds in the corresponding force, will be the moment of that force tending to turn the board upon the point P; and then the principle of the equality of moments, explained in Art. 32, will be observed.

67. Let A B (See fig. page 61,) be a platform with a load C upon it, supported by a chain A D; it is required to determine by construction the tension of the chain, and also the amount and direction of the pressure upon the hinge at B, the weight of the platform being neglected.

Construction. Through the point C draw the vertical line C *o*, intersecting the direction of the chain in the point *o*, join B and *o*, and from a scale of equal parts set off O *n* equal to the units in the weight placed at C; draw *n r* parallel to A D, and *r e* parallel to C *o*, then *n o e r* will be the parallelogram of pressures. Because O is the intersection of two of the forces or pressures; therefore by Prop. 1, Art. 66, the direction of the third force must also pass through O; but this third force must also pass through B,—hence it follows that O B is the direction of the pressure upon the hinge. From the scale of equal parts measure *o r*, and it will give the units of pressure on the hinge, and in like manner *o e* will give the units of pressure tending to break the chain.

Ex. 1. Let B A = 3 ft., B D = 5 ft., B C = 2 ft., and the weight at C = 1½ tons, then it will be found by construction that the pressure on the hinge = ·8 tons, and the tension of the chain = 1·16 tons.

Ex. 2. Required the same as in the last example, when the load is 8 cwts. *Ans.* 4·3 *and* 6·2 *cwts.*

Ex. 3. In the platform of Example 1, the chain is just able to support 24 cwts.; what weight may be placed at C?

Here we first mark off *o e* = 24, then construct the parallelogram *o e r n*, and the units in *o n* = 31 cwts., which is the weight required.

Ex. 4. Required the same as in the last example, when the chain is just able to sustain 32 cwts. *Ans.* 41 *cwts.*

68. A pole O D (see fig. page 60) supported by a cord A D, carries a weight W; it is required to find the tension of the cord, and the pressure on the pole.

Construction. On the line D N mark off D *n* equal to the units of weight in W, draw *n* I parallel to D A, intersecting D O in I, and from I draw I *e* parallel to D *n*; then D *n* I *e* will be the parallelogram of pressures; therefore the units in D *e* will give the tension of the cord, and the units in D I will give the pressure upon the pole.

Ex. 1. Let O D = 50 ft., O A = 20 ft., A D = 60 ft., and W = 25 cwts.; required the tension of the cord, and the pressure on the pole.

Construction. With the three given dimensions construct the triangle A O D, from D let fall the perpendicular D N upon A O produced, from the scale of equal parts mark off D *n* = 25, and construct the parallelogram *n e*, then it will be found that D *e* or the tension = 28 cwts., and D I or the pressure on the pole = 50 cwts.

Ex. 2. Required the same as in the last example, when W = 15 cwts. *Ans.* tension = 16 cwts., and pressure on the pole = 29 cwts.

Ex. 3. If the cord in Ex. 1 is just able to sustain a tension of 35 cwts., what weight may be suspended from D?

Ans. 31 *cwts.*

Here D *e* is taken equal to 35, and then the parallelogram *n e* is constructed.

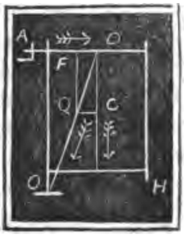
Ex. 4. Required the same as in the last example, when the tension of the cord is 22 cwts. *Ans.* 20 *cwts. nearly.*

Ex. 5. Let O D be a ladder weighing 3 cwts. and having its centre of gravity, *c*, ⅓ from the foot, required the amount and direction of this pressure on the ground, supposing the ladder to be supported by the cord A D.

Construct the triangle A O D as in Ex. 1, take *o c* = ⅓ of 50 = 16·6, through *c* draw R *c* perpendicular to A N, produce R *c* until it intersect A D in a point which we shall call V, join the points V and O, then V O will be the *direction* of the pressure on the ground. Construct the parallelogram of pressures, as in the preceding examples, and the pressure on the ground will be found to be 3 9 cwts.

69. A beam F B is supported by a cord F A; it is required to determine the direction and tension of the cord B H, so that the beam may not change its position.

Construction. Through the centre of gravity, C, draw the vertical line C K, produce A F until it intersect this vertical line in the point K, join K and B, then K B produced will give the *direction* of the cord. Take K I equal to the units in the weight of the beam, and construct the parallelogram of pressures N V, then the units in K V will give the tension of the cord H B.



70. A gate A H is supported by a pin turning in a socket at O, and prevented from falling in the direction A D by a hook and loop at A; it is required to determine the amount and direction of the pressure upon the pin O, and the force tending to draw the hook A from the wall.

Construction. Through the middle, or centre of gravity of the gate, draw the vertical line D C, join the points D and O, then D O will be the direction of the pressure upon the pin. Take D C equal to the units in the weight of the gate, and construct the parallelogram of pressures, C F, then the units in D Q will be the pressure upon the pin, and the units in D F will be the force tending to draw the hook. See Art. 67.

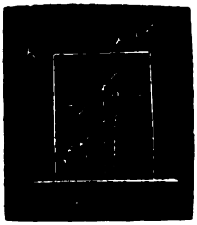
Ex. 1. Let O H = 5 ft., O A = 4 ft., and the weight of the gate = 2 cwts., then it will be found that the pressure upon the pin *o* = 2·35 cwts., and the pressure upon A = 1·25 cwts.

Ex. 2. Required the same as in the last example, when the weight of the gate is 160 lbs.

Ans. Pressure on O = 188 lbs., and pressure on A = 100 lbs.

To determine whether a pillar will stand or fall, when acted upon by a given pressure.

71. Let P be the direction and amount of the pressure, tending to turn the pillar, upon the edge O as a centre; and A V a vertical line passing through the centre of gravity of the pillar, intersecting the line P produced in the point



A; from a scale of equal parts, take A C equal to the units in the pressure P, and from the same scale take A B equal to the units of weight in the pillar; construct the paralellogram of forces A B D C, then by Prop. 2. Art. 66, A D will be the amount and direction of the single force tending to overturn the pillar. If A D produced intersect the base within the edge O, the pillar will stand; and on the contrary if the point of intersection, Q, fall without the base the pillar will fall; but if it intersect at the edge O, then the pillar will just be upon the point of overturning.

It is obvious that if the whole pillar stand, any pillar of a less height will also stand, other things being the same. The point Q is called the point of resistance. Professor Moseley first gave the general equation of the line of resistance for a given structure.

Ex. 1. Let O R = 20 ft., O V = 1 ft., length = 1 ft., R P = 1·5 ft., weight of the material = 100 lbs., P = 200 lbs., and its direction 30° inclined to the horizon.

Here, weight pillar = 20 × 2 × 1 × 100 = 4000 lbs.

Construction. Draw the rectangle R P O, &c.; considering the units on the scale to be thousands, take A B = 4, A C = ·2, complete the parallelogram B C, and the diagonal, A D, produced, shows that the pillar will stand.

Ex. 2. Required the same as in the last example, when P = 400 lbs., and the weight of a cubic foot of the material = 120 lbs. *Ans.* The pillar will fall.

Ex. 3. Required the thrust P, in ex. 1, so that the pillar may just be upon the point of overturning.

Join the points O and A, take A B = 4, and construct the parallelogram B C; measure off A C from the scale, and it will give 250 lbs. nearly, for the pressure required.

Ex. 4. Required the same as in the last example, when the angle which P makes with the horizon is 45°. *Ans.* 300 *lbs.*

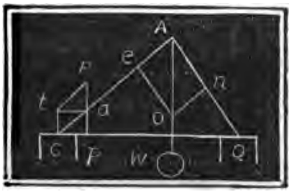
Modulus of Stability.

72. A structure will be more or less stable, according as the point of resistance, Q, is more or less distant from the edge O. Hence the modulus of stability, may be defined to be, the ratio that O Q bears to O V. Thus, if Q were in the middle between O and V, the modulus would be ½; if Q were at V the modulus would be 1, or the most stable possible, under the given conditions; and if Q were at O, the modulus would be O, that is, the structure would be on the point of overturning. Vauban considers that the modulus of a good structure ought to be about ½.

Ex. 1. What must be the pressure P, in Ex. 1, Art. 71, so that the modulus of stability may be ½. *Ans.* 110 *lbs.*

Construction. Bisect O V in the point Q. join Q and A, from a scale of equal parts take A B = 4000, and construct the parallelogram of pressures B C; then the units in A C will give the units of lbs. in the pressure P.

Pressure of Roofs.



73. Let C A and Q A be the rafters of a roof resting upon the side walls at C and Q; it is required to determine the thrust on the points C and Q, the roof being without a tie beam.

Construction. Let us suppose that the weight of the roof is equally distributed over the surface, and that a foot of length of this roof acts upon a foot of length of the side walls. Find the weight of each side of the roof one foot long, and let a weight W, be suspended from A equal to the half sum of these weights, then one half of the whole weight upon the rafter, C A, will act perpendicularly upon the wall at C, the other half acting in the weight W; and in like manner one half of the whole weight upon the rafter, Q A, will act perpendicularly upon the wall at Q, the other half acting in the weight W. Now take A O equal to the units of weight in W, and construct the parallelogram of presperpendicular to B H, then because P D = P H, the pressure on the point P will be due to a column of fluid whose height is P D, and so on to any other point in the side of the vessel. Let us now suppose that the depth of the water, H B, is 3 ft., and the length of the side 4 ft., then the whole pressure upon the side will be equivalent to the pressure, or weight, of a mass of fluid of the form of the wedge, A B H, that is,

$$\text{Pressure} = 3 \times 4 \times \tfrac{3}{2} \times 62{\cdot}5 = 1125 \text{ lbs.}$$

Here we observe that the lbs. pressure upon the side or proposed surface of a vessel is equal to the weight of a column of fluid whose base is the area of the surface, and perpendicular height, the depth of the centre of gravity, or middle point, of that surface.

Ex. 1. Required the pressure on a flood gate whose breadth is 8 ft. and depth 6 ft.

$$\text{Pressure} = 6 \times 8 \times \tfrac{6}{2} \times 62{\cdot}5 = 9000 \text{ lbs.}$$

Ex. 2. The depth of water pressing against an embankment is 9 ft., required the pressure upon each foot of length in the embankment. *Ans.* 2531¼ *lbs.*

Ex. 3. Compare the pressure upon the sides of a cubical vessel filled with water, with the pressure upon the bottom, allowing for the sake of simplicity, that the side of the base is 2 feet.

$$\text{Pressure on the base} = 2 \times 2 \times 2 \times 62{\cdot}5.$$

$$\text{,, sides} = 2 \times 2 \times \tfrac{2}{2} \times 62{\cdot}5 \times 4.$$

$\therefore \dfrac{\text{Pressure on sides}}{\text{Pressure on base}} = 2$, that is, the pressure on the sides is twice that on the base.

Ex. 4. Required the pressure on the staves of a cylindrical barrel filled with water, the diameter of the base being 3 feet, and the perpendicular height 4 feet. *Ans.* 4712 *lbs.*

77. It will readily be seen, that there must be a certain point in the side of a vessel, filled with water, where a single pressure will exactly counterbalance the pressure of the water against the whole side. This point is called the *centre of pressure.* It must obviously lie in the line, D P, passing through

the centre of gravity of the fluid wedge A B H. But by Art. 34, B P $= \frac{1}{3}$ B H, that is, the centre of pressure P, lies at one third from the bottom of the vessel.

If the staves of a barrel be kept together by a single hoop, and if the barrel be filled with water, then the hoop must be placed at one third from the bottom.

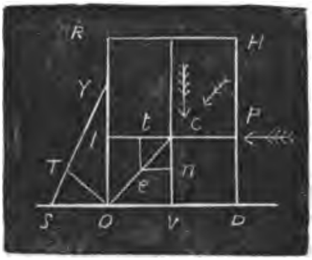
Ex. 1. Required the centre of pressure in Ex. 1, Art. 76.

Ans. 2 *ft.* from the bottom.

Ex. 2. Required the centre of pressure in Ex. 2, Art. 76.

Ans. 3 *ft* from the bottom.

78. An embankment, R D, sustains the pressure of water whose centre of pressure is at P, it is required to determine the conditions of equilibrium, &c., supposing the embankment to turn over upon O as a centre.



Let C V be a vertical line passing through the centre of gravity of the embankment, P the centre of pressure of the water. Draw P I perpendicular to R O, then the product of the pressure of the water by O I, will give the moment of the water, tending to turn the embankment over upon O as a centre; and in like manner, the product of the weight of the embankment by O V, will give the moment of the embankment, tending to turn itself in a direction opposed to the pressure of the water. When these moments are equal, the embankment is upon the point of overturning. If the moment of the water be greater than that of the embankment, the structure will stand, and *vice versâ.* In the following calculations the length of the embankment is taken at one foot, because if it stand for one foot, it will stand for any other length.

Ex. 1. Let O R = 9 ft., O D = 3 ft., and the weight of a cubic foot of the material = 150 lbs., will the embankment stand or fall when the water is at the top?

Surface upon which the water presses $= 9 \times 1$.

Pressure of the water $= 9 \times 1 \times \frac{9}{2} \times 62 \cdot 5 = 2531 \cdot 25$ lbs.

Distance of the centre of pressures from the bottom, or 0 I $= \frac{9}{3} = 3$ ft.

$\therefore$ Moment of the water $= 2531 \cdot 25 \times 3 = 7593 \cdot 75$.

Weight of the embankment $= 3 \times 1 \times 9 \times 150 = 4050$ lbs.

Moment of the embankment $= 4050 \times \frac{3}{2} = 6075$.

Here we find that the moment the water is greater than that of the embankment,—hence the structure will fall.

Ex. 2. Required the same as in the last example, when the height is 12 ft., the thickness 5 ft., and the weight of a cubic foot of the material 120 lbs. *Ans.* The embankment will be just upon the point of overturning.

Ex. 3. What must be the height of the water in Ex. 1, so that the embankment may be upon the point of overturning.

Here we have by equating the moment of the pressure of the water with that of the embankment,

$$\frac{\text{Height}^3}{6} \times 62 \cdot 5 = 6075\text{; and } \therefore \text{ height} = 8 \cdot 3 \text{ ft.}$$

Ex. 4. Required the base, or thickness, of a rectangular embankment, when the height is 15 ft., the weight of a cubic foot of the material is 140 lbs., and the water stands at the brim, so that the structure may be upon the point of overturning.

Proceeding as in the last example, we find,

$$\text{Base}^2 \times 1050 = 35156 \cdot 25\text{, and } \therefore \text{ base} = 5 \cdot 7 \text{ ft.}$$

Ex. 5. Let the embankment have the form of a trapezoid, A H R C, (See fig. to Art. 76,) where A B $=$ 3 ft., B C $=$ 2 ft., R C $=$ 9 ft., and the weight of a cubic foot of the material $=$ 100 lbs.; will the embankment stand or fall, when the water is at the brim?

Let the embankment be divided into two parts, viz. the rectangular part B R, and the triangular part A B H. Now by Art. 34, a vertical line passing through the centre of gravity of the triangle, will cut the base at the distance of 2 ft. from the centre of motion A; and a line through the centre of gravity of B R will cut A C at the distance of 4 ft. from A.

Wt. of A B H $= \frac{3 \times 9}{2} \times 1 \times 100 = 1350$ lbs.

$\therefore$ Moment of A B H $= 1350 \times 2 = 2700$. Similarly, Moment of B R $= 1800 \times 4 = 7200$.

$\therefore$ Moment of the whole embt. $= 2700 + 7200 = 9900$.

Moment of the water $= 9 \times 1 \times \frac{9}{2} \times 62{\cdot}5 \times \frac{9}{3} = 7593$.

Hence it follows that the embankment will stand.

Ex. 6. Required the same as in the last example, when A B = 6 ft., B C = 4 ft., and R C = 15 ft.

Ans. It will stand.

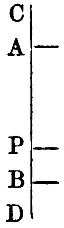
Ex. 7. Required the modulus of stability of the structure in Ex. 1, when the base O D = 5 ft.

Here the wt. of the structure $= 5 \times 1 \times 9 \times 150 = 6750$ lbs.

Pressure of the water = 2531 lbs.

Construction. Draw the rectangle R D, take D P = 3, draw P I parallel to O D, bisect the base in the point V, and draw V C perpendicular to O D, intersecting P I in the point C; from any diagonal scale mark off C *n* = 6750, and C *t* = 2531; construct the parallelogram *n t*,—then the diagonal C *e* produced will intersect the base in a point, which we shall call Q, situated between O and V. The ratio between O Q and O V will be found to be about 13 to 25.

Ex. 8. The breadth of a flood gate is 10 ft., and the depth 6 ft. The hinges are placed at 1 ft. from the respective extremities of the gate. It is required to find the pressure upon the lower hinge.



Here the pressure of the water on each half of the gate $= 6 \times 5 \times \frac{6}{2} \times 62{\cdot}5 = 5625$ lbs.

Let C D be the height of the gate, A and B the hinges, and P the centre of pressure of the water,—then D P $= \frac{6}{3} = 2$; A P $= 5 - 2 = 3$; and B A $= 6 - 2 = 4$. Now since the pressure of the water at P is supported by the hinges A and B, we have by the principle of the lever, Art. 28.

Pressure on B $\times 4 = 5625 \times 3$; $\therefore$ Pressure on B = 4218 lbs.

Ex. 9. Required the same as in the last example, when the breadth of the gate is 8 ft., the depth 7 ft., and the distance between the hinges 4 ft. *Ans.* 4849 *lbs.*

79. Let the embankment be supported by a shore, or stay, S Y; it is required to find the thrust upon this stay when the embankment is upon the point of overturning on the edge O.

Here the moment of the thrust of the stay, added to the moment of the embankment, will be equal to the moment of the pressure of the water. From this equality, the thrust may be readily found.

Ex. 1. Let H D = 12 ft., O D = 1·5 ft., weight of a cubic foot of the material = 130 lbs., S Y = 5 ft., and S O = 3 ft.; required the thrust upon the stay when the embankment is upon the point of overturning on O, the water being at the top.

Wt. embankment = $1·5 \times 1 \times 12 \times 130 = 2340$.

Moment of ,, $= 2340 \times \frac{1·5}{2} = 1755$.

Let O T be the perpendicular upon the stay, then O T = 2·4.

∴ Moment of the thrust on the stay = thrust × 2·4.

Moment of the pressure of the water = 18000.

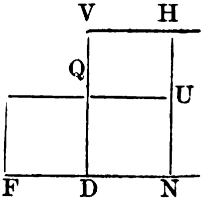
∴ Thrust × 2·4 + 1755 = 18000; and thrust = 6768.

Ex. 2. Required the same as in the last example, when O D = 2 ft. *Ans.* 6200.

Ex. 3. Required the same as in Ex. 1, when the stay is 10 ft. long, and placed with its foot at 8 ft. from O. *Ans.* 3384 *lbs.*

To determine the form of maximum, or equable, strength of an embankment.

80. The embankment, V H N F, will have a maximum strength if the part V U be upon the point of overturning upon the edge Q, at the same time that the portion V H N F is upon the point of overturning upon F, by the pressure of the fluid.



Ex. 1. The height of the embankment, H N = 6 ft., V Q = 3 ft., and the weight of a cubic foot of the material = 120 lbs.; it is required to find the base F N, so that the embankment may be upon the point of overturning on F, at the same time that the part V U is upon the point of overturning on Q, when the water stands at the brim.

Here we must first find the base Q U, so that V U may be upon the point of overturning on Q, by the pressure of the water on H U.

Put x = Q U, then proceeding as in Ex. 4, Art. 78, we find,

$$x^2 \times 180 = 281{\cdot}25\,; \;\therefore\; x = 1{\cdot}3 \text{ ft. nearly.}$$

We have now to find F N, for this purpose let Y = F D, then,

$$\text{Moment F Q} = Y \times 1 \times 3 \times 120 \times \frac{Y}{2}.$$

$$\text{Moment V D N H} = 1{\cdot}3 \times 1 \times 6 \times 120 \times \left(Y + \frac{1{\cdot}3}{2}\right)$$

$$\text{Moment water} = 2250.$$

Now the sum of the two former moments must be equal to the moment of the water, hence we find by reduction,

$$180\,Y^2 + 936Y + 608{\cdot}4 = 2250.$$

Solving this equation we find Y = 1·37, and F N = 1·37 + 1·3 = 2·67 ft.

Ex. 2. Required the same as in the last example, when V Q = 2 ft., and the weight of a cubic foot of the material = 100 lbs. *Ans.* Base of the top part = ·912 ft. nearly, and base at the bottom = 3 ft.

Revetment Walls.

81. A wall sustaining the pressure of earth, or any loose material, is called a revetment wall. The pressure of earth upon a wall is similar to the pressure of water, with only this difference,—that the weight of the material must be reduced by a certain ratio dependent upon the angle of its natural slope. The thrust of earth upon a wall is occasioned by a certain portion, of a wedge shape, tending to break away from the

general mass. Coulomb showed, that the angle which this line of rupture makes with the vertical is one half the angle which the line of natural slope makes with the vertical.

Now when the earth is level at the top, we may always find the pressure of the earth by regarding it as a fluid, having the weight of a cubic foot, equal to the weight of a cubic foot of the earth multiplied by the square of the tangent of half the angle of the natural slope from the vertical. In earth of mean quality the angle of natural slope is about 45°, then the square of the tangent of half this angle will be ·1716, this number, therefore, multiplied by the weight of a cubic foot of the material, gives the weight, in this case, of a cubic foot of the equivalent fluid pressing against the wall.

In like manner it is also found that when the wall is sustained by the pressure of the earth, the weight of the equivalent fluid is about six times the weight of the earth.

Ex. 1. A Revetment wall 40 ft. high, and 10 ft. thick, sustains the pressure of earth of mean quality, having the weight of a cubic foot equal to 100 lbs., it is required to determine whether or not the wall will stand, taking the weight of a cubic foot to be 120 lbs.

Here, wt. c. ft. of the equivalent fluid $= 100 \times \cdot 1716$ lbs.

Pressure of the earth $= 40 \times 1 \times \frac{40}{2} \times 100 \times \cdot 1716 = 13728$.

Moment of the earth $= 13728 \times \frac{40}{3} = 183040$.

Moment of the wall $= 40 \times 10 \times 1 \times 120 \times \frac{10}{2} = 240000$. Hence the wall will stand.

Ex 2. Required the same as in the last example, when the thickness of the wall is 8 ft. *Ans.* It will fall.

Ex. 3. Required the thickness of the wall, in Ex. 1, so that it may be upon the point of overturning.

Let x be put for the thickness, then we have,

Moment of the wall $= 40 \times 1 \times x \times 120 \times \frac{x}{2} = 2400\,x^2$.

Now when the wall is upon the point of overturning, the moment of the wall, must be equal to the moment of the pressure of the earth,

$\therefore 2400 x^2 = 183040$; and $x = 8 \cdot 7$ ft.

Ex. 4. The embankment, in Ex. 1, Art. 78, is supported by earth, of mean quality, 3 ft. in height, and level at the top; will the embankment stand, when the weight of a cubic ft. of the earth is 120 lbs.

Here, wt. of equivalent fluid $= 120 \times 6 = 720$ lbs.

$\therefore$ Moment of the earth $= 3 \times 1 \times \frac{3}{2} \times 720 \times \frac{3}{3} = 3240$. This result being added to the moment of the wall, will give a quantity greater than the moment of the water;—the embankment, therefore, will stand.

In the preceding examples, the earth is supposed to be level at the top, but when the earth has its natural slope, a similar method of calculation will apply. When the wall resists the pressure of the earth, we must take $\frac{1}{8}$ of the weight of the earth, for the weight of the equivalent fluid; and on the contrary, when the earth resists the pressure of the wall we must take $\frac{1}{3}$ the weight of the earth for the weight of the equivalent fluid. See Art. 324 and 328, of Moseley's Engineering.

Ex 5. Required the same as in Ex. 1, when the earth has its natural slope, and the thickness of the wall is 8 ft.

Here, the wt. of a c. ft. of the equivalent fluid $= 100 \times \frac{1}{8}$ lbs.

$\therefore$ Moment of the earth $= 40 \times 1 \times \frac{40}{2} \times 100 \times \frac{1}{8} \times \frac{40}{3}$ $= 133333$.

Moment of the wall $= 153600$.

Hence it follows that the wall will stand.

Ex. 6. A Revetment wall is 30 ft. high and 6 ft. thick. On one side of it earth of mean quality is sustained level with the top, and on the other side, the earth has its natural slope, and rises to the height of 5 ft.; will the wall stand or fall, supposing that the weight of a cubic foot of the earth to be 120 lbs., and that of the wall 130 lbs.?

In this case, the moment of the earth, which rises to the level of the wall, is opposed by the sum of the moments of the wall, and the earth having a natural slope.

Weight of the wall $= 23400$ lbs.

∴ Moment of the wall, $= 23400 \times \frac{6}{2} = 70200$.

Wt. of a c. ft. of the equivalent fluid,—with respect to the sloping earth $= 120 \times \frac{1}{2} = 60$ lbs.

Pressure of this earth $= 5 \times 1 \times \frac{5}{2} \times 60 = 750$ lbs.

Moment " $= 750 \times \frac{5}{3} = 1250$.

Total moments sustaining the wall $= 70200 + 1250 = 71450$.

Proceding as in Ex. 1, we have,

Moment of the level earth $= 92664$.

As the latter moment is greater than the sum of the two former, we conclude that the structure will fall.

Ex. 7. Required the same as in the last example, when the height of the wall is 24 ft. *Ans.* It will stand.

Ex. 8. What must be the thickness of the wall in Ex. 6, when the wall is upon the point of being overturned?

Ans. 6·8*ft.*

Ex. 9. What must be the height of the sloping earth in Ex. 6, when the wall is upon the point of being overturned?

Ans. 13*ft.*

Floating Bodies.

82. When a body floats it is sustained by the upward pressure of the fluid; and as there is an equilibrium of pressures, the upward pressure must be equal to the gravity of the floating body. But this upward pressure would just support a volume of fluid equal to that which is displaced, therefore the weight of the floating body must be equal to the weight of the displaced fluid.

Ex. 1. A barge (supposed for the sake of simplicity to be of a rectangular shape) is 10 ft. long, 5 ft. broad, and 4 ft. deep, outside measure. The thickness of the planking is 2 inches, and the weight of a cubic foot of the timber is 50 lbs. To what depth will the barge sink when loaded with 4 tons?

Content of the exterior solid $= 10 \times 5 \times 4 = 200$.

" interior " $= 9\frac{2}{3} \times 4\frac{2}{3} \times 3\frac{5}{6} = 172{\cdot}92$.

∴ Timber in the barge $= 200 - 172{\cdot}92 = 27{\cdot}07$ c. ft.

Wt. Timber in the barge = 27·07 × 50 = 1353 lbs.

Wt. displaced water = 10 × 5 × depth × 62·5 = 3125 × depth.

∴ 3125 × depth = 1353 + 4 × 2240; and depth = 3·3 ft.

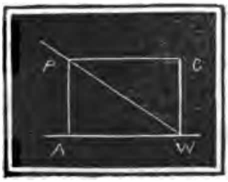
Ex. 2. Required the same as in the last example, when the thickness of the planking is 3 in. *Ans.* 3·5*ft.*

Ex. 3. What load will just sink the barge in Ex. 1. *Ans.* 4·9 *tons.*

Ex. 4. What load will sink the barge in Ex. 1, to the depth of 3 ft. *Ans.* 3·58 *tons.*

Ex. 5. How far will the barge, in Ex. 1, sink when empty? *Ans.* ·433*ft.*

Limiting angle of resistance.



83. Let W be a material particle acted upon by a pressure, P, in the direction of and equal in magnitude to P W. Complete the rectangle, A C, then, by Art. 66, C W will be the pressure upon the plane, and A W that part of the force, P, which tends to give motion to the particle along the plane. Put a for the angle P W C, then we have,

$$\text{Pressure on the plane} = \text{C W} = \text{P} \cos a.$$

$$\therefore \text{Resistance of friction} = f\,\text{P} \cos a.$$

Effective pressure tending to move the particle in opposition to friction = A W = P $\sin a$.

Motion will or will not take place, according as this pressure is greater or less than the resistance of friction; and when motion is upon the point of taking place, the one must be equal to the other, and then the angle a is called the limiting angle of resistance; in this case, therefore, we have,

$$f\,\text{P} \cos a = \text{P} \sin a;$$

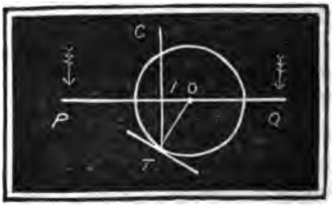
$$\therefore f = \frac{\sin a}{\cos a} = \tan a.$$

This result shows that the coefficient of friction is equal to the tangent of the limiting angle of resistance. If the pressure P, be applied within this angle, then no motion can take place, however great that pressure may be; and on the contrary, if the pressure be applied without this angle, then motion will take place, however small that pressure may be. Professor Moseley makes this property the basis of his theory of machines.

Ex. 1. If the coefficient of friction be ·7, what is the limiting angle of resistance? *Ans.* 35°.

Ex. 2. Will the foot of the ladder, in Ex. 5, Art. 68, slip supposing the angle of friction to be 36°. *Ans.* It will not slip.

Equilibrium of the lever, and the wheel and axle, taking the friction upon the axis into account.



84. Let P O Q be a lever having the circular axis C T, turning within a circular socket. In order that motion should take place in the direction of the pressure P, the resultant of the two pressures, P and Q, must fall on the left side of the centre O of the axis, and, by Art. 83, this resultant, C T, must be inclined to O T, at the limiting angle of resistance. Taking T, therefore, as the centre of motion, we have by the equality of movements for the state bordering on motion, P × P I = Q × Q I.

Ex. 1. Let P O = 14 in., Q O = 12 in., O T = 3 in., P = 1 lb., and angle O T I = 30°.

Here O I = *sin* 30° × 3 = $\frac{3}{2}$; P I = 14 — $\frac{3}{2}$ = $\frac{25}{2}$; Q I = 12 + $\frac{3}{2}$ = $\frac{27}{2}$,

$\therefore$ 1 × $\frac{25}{2}$ = Q × $\frac{27}{2}$, and Q = ·92. lbs.

Ex. 2. Required the same as in the last example, when angle O T I = 40°. *Ans.* ·86 lbs.

Ex. 3. Let P O = 15, Q O = 8, O T = 2, P = 2 lbs., and the limiting angle of resistance = 35° *Ans.* Q = 3 *lbs.*

Ex. 4. Required the same as in Ex. 1, when the angle which the pressure P makes with the lever is 30°, and that of the Q is 35°.

Construction. Produce the pressures until they intersect in a point which we shall call C, and join the points C and O. Now we have to draw the resultant of the forces P and Q, so as to make the angle O T C = 30°; for this purpose, describe a circle upon the chord C O, so as to contain the vertical angle = 30°. Let this circle intersect the circumference of the axis in the point T, join T and C, on the line C P take off a space equal to the units in the pressure P, then construct the parallelogram of pressures, and the units in the side lying on the line C Q will give the pressure Q = ·8 lbs.

Ex. 5. Required the same as in the last example, when the limiting angle of resistance is 35°. *Ans.* Q = ·77 *lbs.*

85. Let O P be the radius of a wheel, and O Q the radius of an axle; put W for the weight of the wheel and axle, then, as this weight acts through the centre O, we have,

$$P \times PI = Q \times QI + W \times OI.$$

Ex. 1. The radius of the wheel is 24 in., the axle 3 in., and the axis 1 inch; required Q, when the power applied to the wheel is 60 lbs., the weight of the wheel and axle 40 lbs., and the limiting angle of resistance 30°.

Here $OI = \frac{1}{2}$; $PI = 24 - \frac{1}{2} = 23\frac{1}{2}$; $QI = 3 + \frac{1}{2} = 3\frac{1}{2}$; $P = 60$ lbs.; and $W = 40$ lbs.,

$\therefore 60 \times 23\frac{1}{2} = Q \times 3\frac{1}{2} + 40 \times \frac{1}{2}$; hence $Q = 397$ lbs.

Ex. 2. Required the same as in the last example, when the limiting angle of resistance is 35°. *Ans.* 386 *lbs.*

To determine generally the force of Traction.

86. Let W (see fig. Art. 83) be a heavy body drawn along the horizontal plane A W, by the force of the traction P, acting in the direction and with the magnitude W P. This force may be resolved into W A and W C, the former force just

overcomes the resistance of friction, the latter tends to reduce the pressure of the body on the plane; hence we have, putting b for the angle P W A,

Pressure W on the plane $= W - P \sin b$.

$\therefore$ Resistance of friction $= f\,(W - P \sin b.)$

But when motion is about to take place, this resistance is equal to the force W A or $P \cos b$.

$\therefore P \cos b = f\,(W - P \sin b.)$

$$\therefore P = \frac{fW}{\cos b + f \sin b} \;\ldots (1)$$

Precisely in the same way the traction may be found when the body is moved upon an inclined plane.

Ex. 1. A stone weighing 2 cwts. is drawn along a horizontal plane, by a cord inclined at an angle of 35°; required the force necessary to move the stone supposing the coefficient of friction to be ·7. *Ans.* By substituting the given quantities in the formula (1), we find P = 1·14 cwts.

Ex. 2. Required the same as in the last example, when the angle is 40°. *Ans.* 1·19 *cwts.*

To find the least traction.

87. For f substitute its value given in Art. 83, then by an easy reduction, we find,

$$P = \frac{W \sin a}{\cos\,(b - a)} \;\ldots (1)$$

Now P will be the least possible when the denominator of this expression is the greatest possible, which will obviously obtain when $a - b = 0$, or $a = b$, that is, when the angle of traction is equal to the limiting angle of resistance. In this case, therefore, we have, $P = W \sin a \;\ldots (2)$.

Ex. 1. Required the least traction in Ex. 1, Art. 86, taking $a = 40°$.

Here by eq. (2), $P = 2 \times {\cdot}6427 = 1{\cdot}285$ cwts.

Ex. 2. Required the least traction, when the weight of the mass is 9 cwts., and the rubbing surfaces are soft calcareous stone upon the same, giving 36° 30′ for the limiting angle of resistance. *Ans.* 5·35 *cwts.*

Proposition.

88. If a heavy body be moved upon a rubbing surface, lying between two given points, by a pressure acting parallel to the surface, the work is always the same whatever may be the form of the surface.

Let W (see fig. page 70,) be a body moved on the plane A C, then by the resolution of pressures,

The pressure of W on the plane $= W \cos A$.

$\therefore$ Resistance of friction $= f W \cos A$.

$\therefore$ Total work on the plane A C $= f W \cos A \times AC + W \times CB. = f W \times AB + W \times CB$.

Now this expression is *independent of the inclination of the plane*, it being in fact, the work expended in moving the body over the horizontal distance A B, added to the work due to gravity in elevating the body the vertical space B C. As a curved surface may be regarded as being made up of an infinite number of straight planes, therefore the work upon the whole curve will be equal to the work done upon the horizontal projection of the curve, added to the work done in opposition to gravity. When the body descends the surface, the work of gravity becomes minus. This total work is, obviously, independent of the nature of the curve. In general it may be shown, that whatever may be the zigzag course pursued by the body, the work will always be equal to the work on the projection of this curve added to the work due to gravity. This property is an example of what the author proposes to call, the principle of the conservation of work.

Observation. When the inclination of the plane is small, the horizontal distance may be taken equal to the length of the plane. See Art. 11.

Ex. 1. What work would a horse perform in drawing a load of 1 ton to the *horizontal* distance of $\frac{1}{4}$ mile, up a curved incline whose total elevation is 200 ft., the coefficient of the friction of the road being $\frac{1}{20}$.

Work on the horizontal line $= \frac{2240}{20} \times \frac{5280}{4}$.

Work due to gravity $= 2240 \times 200$.

$\therefore$ Total work $= 147840 + 448000 = 595840$.

Ex. 2. Required the work, when the elevation is 80 ft., and the coefficient of friction $\frac{1}{12}$. *Ans.* 425600.

Ex. 3. Required the work in the last example, when the body is drawn down the surface. *Ans.* 67200.

89. If the body W be upon the point of descending the plane by the force of gravity alone, then, the work necessary to move the body is nothing, and in this case we have,

$f \text{W} \times \text{A B} - \text{W} + \text{C B} = 0$; whence $f = \tan \text{A}$.

This result is the same as that obtained in Art. 83.

Parallelogram of pressures proved on the principle of work.

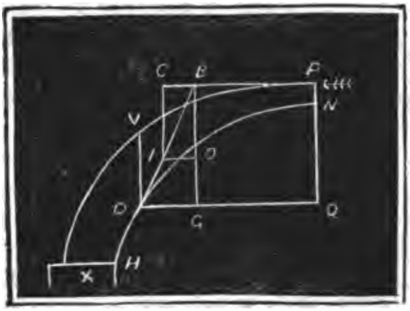
90. Let A C and B D be two inclined planes intersecting each other at right angles in the point W. Let a body W be placed between the two planes; then the force with which the body tends to descend the plane D B, will be equal to the pressure which it exerts upon the plane C A; and the tendency to descend the plane C A will be equal to the pressure exerted on the plane D B. Let a vertical line W H be drawn through the body, and take W H equal to the units of weight in W; from H let fall H S perpendicular to the plane A C, and H V perpendicular to the plane B D, then putting A and B for the inclinations of the planes A C and B D, we have, by trigonometry,

W S $=$ W *sin* A $=$ (by Art. 42) tendency down the plane A C.

Similarly W V $=$ tendency down the plane B D.

Now these forces, W S and W V, are generated by the gravity of the body, or a force represented by the diagonal W H. Having thus established the proposition for the case of the rectangle, it may be very readily extended to the general form of the parallelogram.

Equilibrium of the Arch.



91. When the centres of an arch are taken away, the crown almost invariably sinks; this occasions the joint at the crown to open at its lower edge, and at the same time a certain portion, D V P N, of the arch to turn upon D as a centre thereby producing a rupture, or opening, in the exterior edge at this point. The same effect will take place in the other half of the arch. These two equal portions which thus tend to break away from the general mass, exert a horizontal pressure along the line P C, thereby tending to cause the walls of the structure to turn on their outer edges. The arch will undergo a rupture at that point where the portion, D V P N, so breaking away, will produce the greatest horizontal thrust; for this point must, obviously, be the yielding part of the arch.

To find the point of rupture. Let B G be a vertical line, passing through the centre of gravity of the mass D P, and intersecting P C in the point B; join the points B and D, take B O equal to the units of weight in the mass D P, and construct the rectangle O C, then the units in B C will be the horizontal thrust, if it be greater than any other so determined, and the point D will be the point of rupture. This method is due to Professor Moseley. M. G. Lame and E. Clapeyron found that the resultant D B forms a tangent to the intrados of the arch, when the line of rupture is assumed to follow the vertical line D V. This elegant property gives a very easy method for finding the point of rupture; for if upon constructing the figure, as already described, it is found that

B D *touches* the curve, then D will be the point of rupture. The most troublesome part of this calculation, consists in finding the centre of gravity of the mass D V P N. This has hitherto been done by dividing the arch into a series of small parts, finding the centre of gravity of each part by construction, and then by equating the sum of the moments of the parts, with the moment of the whole. The calculation thus conducted is exceedingly operose. To shorten this labour the author has given the following theorem.

Theorem. If D Q be divided into n equal parts, and lines be drawn from the points of devision perpendicular to D Q, cutting the intrados and extrados; let X and Y be put for D V and N P, the extreme ordinates respectively, and A, B, C, &c., for the first, second, third, &c. intermediate ordinates, or vertical distances between the intrados and extrados, then putting d for the common distance between the ordinates,

$$DG = d\left\{X + (3n - 1)Y + 6(A + 2B + 3C + \&c.)\right\} \div 3\left\{X + Y + 2(A + B + C + \&c.)\right\}$$

Cor. If that part of the divisor which is within the brackets be multiplied by $\frac{d}{2}$ it will give the area of D V P N.

Having ascertained the horizontal thrust, B C, we must now find the centre of gravity of the whole mass, including the semi-arch X P N H, with its load if any, and the pier H X; then supposing the pier to turn upon its outer edge, if the moment of this mass exceeds the moment of the horizontal thrust, the structure will stand, and *vice versá*. See Art. 71.

Ex. 1. The radius of a semi-circular arch is 11 ft., the thickness at the crown 18 in.; the mason work is built level with the crown, and the weight of a cubic foot of the material is 120 lbs.; required the point of rupture, and the horizontal thrust, taking 1 foot length of the arch. *Ans.* 65° from the crown; and thrust 1900 lbs. nearly.

Here constructing the figure on the scale of an inch to the foot, and dividing D Q into 6 equal parts, we find X = 7·8, A = 5·4, B = 3·8, C = 2·8, D = 2, E = 1·6, Y = 1·5, and $n = 6$, $d = \frac{4}{3}$. Then by the formula, we readily find, D G = 3·53 ft., and weight D V P N = 4000 lbs. Take B O = 4000 lbs,, and construct the parallelogram O C, then B C = 1900 lbs. nearly; and as B D forms a tangent to the curve, the point D will be the point of rupture.

Ex. 2. If the piers in the last example be 4 ft. thick, and their height 28·5 ft. measured to the level of the crown; will they stand or fall?

Here the weight of the whole semi-arch = 5100 lbs.

The distance of its centre of gravity from the outer edge of the pier = 7·6 ft. Weight of the whole pier = 4 × 1 × 28·5 × 120 = 13680 lbs. Hence, we find, moment of the pier = 13680 × 2; moment of the whole semi-arch = 5100 × 7·6; and moment of the horizontal thrust = 1900 × 28·5. The sum of the two first moments being greater than the moment of the thrust, it follows that the pier will stand. This may also be verified by construction.

Ex. 3. If the pier be 3 ft. thick, will the structure stand?

Ans. It will fall.

Ex. 4. What must be the thickness of the pier, so that it may be upon the point of overturning? *Ans.* 3·3 *ft.*

Ex. 5. In a semi-circular arch the radius of the intrados is 8, that of the extrados 9·5 ft., and the thickness at the crown is 9 inches; required the point of rupture.

Ans. 58° from the crown.

RAILWAY CUTTINGS AND EMBANKMENTS.

92. In order that a railway cutting or embankment may stand the action of the weather, and other causes which tend to disintegrate portions from the general mass, the sides must have an inclination corresponding to the natural slope of the particular material. Thus when the material is soft clay, the sides must fall away about 2 feet for every foot in the vertical line, and then the slopes are said to be 2 to 1; for gravel the

slopes may be about $1\frac{1}{2}$ to 1; for chalk about 1 to 1; and for unstratified rocks the sides may be vertical, or 0 to 1. The vertical heights of a cutting are usually taken at 1 or 2 chains apart. If we conceive vertical sections to be made through the extreme edges of the rail, the solid will be divided into three parts,—the central portion will be a prism, and the slopes lying on each side will, in general, have the form of a frustrum of a cone. The following symmetrical formula for finding the content of a cutting, or embankment, is a useful modification of that given by Professor Moseley.

Let g be the breadth of the rail; a, b, c, d, &c. the perpendicular heights taken at q feet apart; and p to 1 the ratio of the slopes, then

The content of the central part $= \frac{gq}{2}\left\{a + v + 2(b + c + d + \&c.)\right\}$; where a and v are the extreme ordinates, and $b + c + d + \&c.$ are the intermediate ones.

Content of the two slopes $= \frac{pq}{3}\left\{(a + b)^2 + (b + c)^2 + (c + d)^2 + \&c. - (ab + bc + cd + \&c.)\right\}$

Ex. 1. Required the solid content of a cutting or embankment, whose heights, taken at 1 chain apart. are 6, 3, 4, 5, 7, and 2 feet; the width of the rail 30 ft., and the slopes 3 to 1.
Ans. 67782 *c. ft.*

Content of the central part $= \frac{30 \times 66}{2}\left\{6 + 2 + 2(3 + 4 + 5 + 7)\right\}$

Content of the slopes $= \frac{3 \times 66}{3}\left\{(6 + 3)^2 + (3 + 4)^2 + (4 + 5)^2 + (5 + 7)^2 + (7 + 2)^2 - (6 \times 3 + 3 \times 4 + 4 \times 5 + 5 \times 7 + 7 \times 2)\right\}$

Ex. 2. The heights taken at 2 chains apart, are 4, 5, 7, 3, 6, and 0 feet; the width of the rail is 32 ft., and the slopes are 3 to 1; required the content of the cutting.
Ans. 143088 *c. ft.*

Ex. 3. Required the content of an embankment, whose heights, taken at 1 chain apart, are 0, 8, 9, 2, 1, and 0 feet; the width of the rail 30, and the slopes 2 to 1. *Ans.* 40128 *c.ft.*

Ex. 4. Required the content of a cutting, whose heights, taken at 2 chains apart, are 9, 11, 12, 8, 22, and 6 ft.; width of the rail 30 ft., and slopes $1\frac{1}{4}$ to 1. *Ans.* 370370 *c. ft.*

Ex. 5. The heights taken at 2 chains apart, are 5, 6, and 4 ft., the width of the rail 30 ft., and the slopes 2 to 1; required the content. *Ans.* 56276 *c.ft.*

Ex 6. The heights taken at 2 chains apart, are 0, 10, 30, 40, 0 ft., the width of the rail 30 ft., and the slopes 4 to 1. *Ans.* 1496000 *c.ft*

Ex 7. If the material in the last example is transported to the mean distance of 5 miles with the speed of 10 miles per hour with a full load of 200 c. ft., and returns with the empty waggon with the speed of 20 miles; how many men of each sort will it take to keep two engines at work, and what will be the cost of the excavation exclusive of that of the engine, allowing each workman 3*s*. per day of 10 hours, and that the material requires 2 pickmen to 3 shovellers? *Ans.* $10\frac{2}{3}$ shovellers, $7\frac{1}{9}$ pickmen, and cost = £748.

Ex. 8. Required the cost, &c., as in the last example, when the heights of the cutting taken at 1 chain apart, are 0, 20, 30, 60, 40, and 0 ft., the width of the rail 30 ft., and the slopes 1 to 1. *Ans.* Content of the cutting = 688600 c. ft., and cost = £344. 6*s*. 0*d*.

EXAMPLES FOR PUPILS.

The number prefixed to each set of examples refers to the article in the text, where the principle, involved in the questions, is explained.

2.

1. Required the work in raising 7 lbs. to the height of 15 ft. *Ans.* 105.

2. What work will it take to raise 3 cwts. to the height of 5 ft. *Ans.* 1680.

3. What work will it take to raise a cubic foot of water to the height of 36 ft. *Ans.* 2250.

4. If the height be 24 ft., what will then be the work? *Ans.* 1500.

5. To what height must I raise 7 lbs. so as to perform 35 units of work? *Ans.* 5 *ft.*

6. What work will it take to raise 6 ft. of water to the height of 8 ft. *Ans.* 3000.

3.

1. What must be the H. P. of an engine to pump 87 ft. of water per min., from the depth of 60 fathoms? *Ans.* 59·3.

2. If the depth be 25 fathoms, what will then be the H. P.? *Ans.* 24·7.

3. How many feet of water, will an engine of 50 H. P., raise per min., from a depth of 90 fathoms? *Ans.* 48·8.

4. If the H. P. be 10, how many feet of water will then be raised? *Ans.* 9·7.

5. How many tons of coals will an engine of 3 H. P. raise per hour, from a pit whose depth is 120 fathoms? *Ans.* 3·6.

6. If the depth of the pit be 180 fathoms, how many tons of coals will then be raised? *Ans.* 2·4.

7. What must be the H. P. of an engine in order to raise 14 tons of coals per hour, from a pit, whose depth is 80 fathoms? *Ans.* 7·6.

8. If the depth be 60 fathoms, what will then be the H. P.? *Ans.* 5·7.

9. From what depth, will an engine of 20 H. P., pump 40 ft. of water per min. *Ans.* 44 *fa.*

10. If the H. P. be 15, what will then be the depth? *Ans.* 33 *fa.*

11. In what time will an engine of 40 effective H. P. pump 4000 ft of water to the mean height of 60 fathoms? *Ans.* 1 *h.* 8 *m.*

12. If the mean height be 15 fathoms, what will then be the time? *Ans.* 17 *min.*

13. A winding engine is observed to raise a tub of coals, weighing 4 cwts., in 4 min. from a pit whose depth is 80 fathoms; required the H. P. of the engine. *Ans.* 1·6

14. If the depth be 100 fathoms, what will then be the H. P.? *Ans.* 2.

5.

1. How many sacks of flour, each weighing 2½ cwts., will a man carry on his back, in a day of 6 hours, to the height of 30 ft. *Ans.* 48. See table, page 9.

2. If the height be 25 ft., how many sacks will he then carry? *Ans.* 58.

3. In what time will a labourer throw 1692 c. ft. of earth, to the mean height of 5 ft., allowing 100 lbs. for the weight of each cubic foot? *Ans.* 3 *days*

4. If there are 5076 c. ft. of earth, what will then be the time? *Ans.* 9 *days.*

5. How many tons of coals will a man, turning a handle as in the windlass, raise in a day of 8 hours, from a pit whose depth is 20 fathoms? *Ans.* 4·6.

6. If the depth of the pit be 28 fathoms, how many tons will he then raise? *Ans.* 3·3.

7. If the man, in Ex. 5, works with a cord and pulley, how many tons will he then raise? *Ans.* 2 09

8. In what time will a man pump 400 c. ft. of water to the mean height of 30 ft., allowing that he can perform, in this case, 2600 units of work per min.? *Ans.* 4 *h.* 48 *min.*

9. If the mean height be 10 ft., what time will he then take? *Ans.* 1 *h.* 36 *min*

10. How many feet of water will the man, in Ex. 8, pump in a day of 8 hours from the depth of 20 fathoms? *Ans.* 166·4.

11. A ram weighs 5 cwts. and has a mean fall of 12 ft., how many strokes will 2 men, working with a handle, give to the ram in a day of 8 hours? *Ans.* 371·4.

12. If the mean height of the fall be 16 ft., how many strokes will then be given? *Ans.* 278·5.

7.

1. What pressure would just move a train of 45 tons on a railway whose friction is 6 lbs. per ton? *Ans.* 270.

2. What work will be due to friction in moving this train over the space of 9 ft.? *Ans.* 2430.

3. A train of 80 tons moves at the uniform speed of 20 miles per hour, on a level rail whose friction is 6 lbs. per ton; required the H. P. due to friction. *Ans.* 25·6.

4. If the weight of the train be 60 tons, what will then be the H. P.? *Ans.* 19·2.

5. Required the H. P., when the weight of the train is 100 tons, the speed 15 miles, and the friction 7 lbs. per ton. *Ans.* 28.

6. If the friction be taken at 5½ lbs. per ton, what will then be the H. P.? *Ans.* 22.

7. When the H. P. of an engine is 30, the speed 40 miles, and the friction 7 lbs. per ton, what must be the weight of the train? *Ans.* 40·1 *tons.*

8. If the H. P. be 25, what will then be the weight of the train? *Ans.* 33·4 *tons.*

9. At what rate, in miles per hour, will a train of 80 tons be drawn by an engine of 70 H. P., when the friction is taken at 6 lbs. per ton, the resistance of the atmosphere being neglected? *Ans.* 54·6 *miles.*

10. If the weight of the train be 100 tons, what will then be the speed? *Ans.* 43·7 *miles.*

11. What will be the speed, when the weight of the train = 120 tons, the H. P. = 60, and the friction 6 lbs. per ton. *Ans.* 31·2 *miles.*

12. If the friction of the rail be taken at 8 lbs. per ton, what will then be the maximum speed? *Ans.* 23·4 *miles.*

13. In what time will an engine of 100 H. P., moving a train of 90 tons, complete a journey of 80 miles, supposing the friction to be 7 lbs. per ton? *Ans.* 1·34 *hours.*

14. If the weight of the train be 45 tons, what will then be the time? *Ans.* ·67 *hours.*

15. If a horse draw a load of 1 ton to the distance of 260 ft., on a road whose friction is $\frac{1}{20}$ of the load, what work will he do? *Ans.* 29120.

16. If the coefficient of friction be $\frac{1}{30}$, what work will then be done? *Ans.* 19413.

9.

1. Required the H. P. of an engine to cut 4000 sq. ft. of oak planking in a day of 10 hours long? *Ans.* 5·8.

2. If there are 5000 sq. ft., how many H. P. will then be required? *Ans.* 7·2.

3. How many feet of plank will an engine of 3 H. P. cut in a day of 12 hours long? *Ans* 2457.

4. If the engine works for 10 hours per day, how many feet will then be cut? *Ans.* 2048.

5. In what time will an engine of 9 H. P. cut 6000 sq. ft. of oak timber? *Ans.* 9·76 *hours.*

6. Required the time when the H. P. = 10. *Ans.* 8·78 *hours.*

11.

1. If a train of 60 tons moves up a gradient, having a rise of 1 foot in 100 feet, with the uniform speed of 20 miles per hour, what will be the H. P. due to gravity alone? *Ans.* 71·6.

2. Required the H. P., when the speed is 15 miles. *Ans.* 53·7.

3. If the friction of rail, in Ex. 1, be 6 lbs. per ton, what will be the H. P. due to friction and gravity? *Ans.* 90·8.

4. A train of 75 tons ascends an incline, having a rise of $\frac{1}{8}$ in 100, with the uniform speed of 40 miles per hour, what is the H. P. due to friction and gravity, assuming the friction to be 6 lbs. per ton? *Ans.* 70·4.

5. If the rise be $\frac{1}{3}$ in 100, what will then be the H. P.?
Ans. 107·7.

6. Required the H. P., in Ex. 4, when the train descends the incline? *Ans.* 25·6.

7. A train of 75 tons descends a gradient, having a rise of $\frac{1}{8}$ in 100, with the uniform speed of 50 miles per hour, what must be the H. P., assuming friction to be 7 lbs. per ton?
Ans. 42.

8. What will be the H. P., when the rise of the rail is $\frac{1}{4}$ in 100? *Ans.* 14.

9. What must be the H. P. in the last example, when the train ascends the incline? *Ans.* 126.

10. If the weight of the train in Ex. 7, be 60 tons, what will then be the H. P.? *Ans.* 33·6.

11. What work will a horse perform in drawing 24 cwts. up a hill of 1 mile, and having a rise of 2 in 100, when the friction of the road is $\frac{1}{12}$ of the load? *Ans.* 1466572.

12. What will be the work when the load is 16 cwts.?
Ans. 077714.

13.

1. Fifty thousand c. ft. are to be conveyed to the horizontal distance of 720 ft.; the material requires 1 pickman to 3 shovellers; required the cost, allowing the shovellers and barrowmen 3*s.* each per day, and the pickmen 3*s.* 4*d.*
Ans. £110. 11*s.* 1$\frac{1}{4}$*d.*

2. What will be the cost, when there are 30,000 c. ft. of earth? *Ans.* £66. 6*s.* 8*d.*

3. Required the cost in Ex. 1, when the earth is first to be raised by ramps to the mean height of 8 ft.
Ans. £128. 11*s.* 1$\frac{1}{4}$*d.*

4. An excavation is to be made 200 ft. long, 40 ft. broad, and 25 ft. high; the material is to be transported by means of barrows, to the mean horizontal distance of 480 ft.; the material requires 4 pickmen to 3 shovellers; what will be the cost, allowing 2*s.* 6*d.* per day for each labourer?
Ans. £316. 13*s.* 4*d.*

5. Required the cost, when there are 80,000 c. ft. of earth. *Ans.* £126. 13*s*. 4*d*.

6. Required the cost, when the content of the earth = 200,000 c. ft., the mean distance = 380 ft., the wages of each labourer = 2*s*. 6*d*. per day, and 3 pickmen are required to 2 shovellers. *Ans.* £283. 6*s*. 8*d*.

7. If the content of the earth = 5000 c. ft., what will then be the cost? *Ans.* £7. 1*s*. 8*d*.

8. How many men of each sort will it take to complete the excavation of Ex. 4, in 40 days?

Ans. 10 shovellers, 40 barrowmen, and 13½ pickmen.

9. What will be the cost, in Ex. 4, when the length of the excavation is 140 ft.? *Ans.* £221. 13*s*. 4*d*

10. 200,000 c. ft. of earth is first to be raised to the mean height of 13 ft. by means of ramps, and then conveyed to the horizontal distance of 480 ft.; required the cost, when the material requires 4 pickmen to 3 shovellers, and the wages of each labourer is 2*s*. 6*d*. per day. *Ans.* £414. 3*s*. 4*d*

11. 90,000 c. ft. of earth are to be raised to the mean height of 10 ft. by means of ramps; how many men will it take to complete the work in 20 days, supposing the material to take 3 pickmen to 5 shovellers. *Ans.* 27·9.

12. If there are 60,000 c. ft. of earth, how many men will then be required? *Ans.* 18·6.

14.

1. It has been found that the total resistance of the air to a train of 8 coaches is 50 lbs. when the train moves at 10 miles per hour. Required the H. P. due to the resistance of the air alone, when the train moves at the rate of 40 miles.

Ans. 85·3.

2. If the train moves at the rate of 20 miles, what will then be the H. P.? *Ans.* 10·6.

3. The resistance of the air to a train of four coaches is 15 lbs., when the train moves at the rate of 10 miles, the weight of the train is 20 tons, the friction 6 lbs. per ton, and the speed 20 miles; required the H. P. *Ans.* 9·6.

4. If the speed be 30 miles, what will then be the H. P.? *Ans.* 24.

5. A train of 90 tons moves upon a level rail, whose friction is 6½ lbs. per ton, with the uniform speed of 40 miles per hour. Now the resistance of the air to a train of this length is 40 lbs. when the speed is 10 miles. Required the H. P. of the engine. *Ans.* 130·6.

6. If the speed in the last example be 50 miles, what will then be the H. P.? *Ans.* 211.

7. Required the H. P., as in Ex. 5, taking the weight of the train = 60 tons, the friction = 6 lbs., the speed = 60 miles, and the resistance of the air = 35 lbs. *Ans.* 259.

8. What must be the H. P. of the engine, when the weight of the train = 70 tons, the friction = 6 lbs. per ton, the speed = 30 miles, and the train undergoes a total atmospheric resistance of 400 lbs. at this speed? *Ans.* 65·6.

15.

1. How many cubic feet of water will an engine of 8 H. P. raise per hour, from a pit whose depth is 160 fathoms, supposing the modulus of the pump to be ⅔? *Ans.* 176.

2. If the depth of the pit be 200 fathoms, how many cubic feet of water will then be raised? *Ans.* 140·8.

3. What must be the effective H. P. of the engine, working the pump in Ex. 1, so as to raise 2·2 c. ft. of water per min? *Ans.* 6.

4. If the water raised per min. be 1·65, what must then be the effective H. P.? *Ans.* 4·5.

5. In what time will the engine in Ex. 1, pump 44 c. ft. of water? *Ans.* 15 *min.*

6. What must be the effective H. P. of an engine working for 8 hours per day, to supply 800 families with 20 c. ft. of water each per day, supposing the water to be raised to the mean height of 60 ft., and the modulus of the pump to be ⅔? *Ans.* 5·68.

7. If water supplied to each be 9 c. ft., what must then be the H. P.? *Ans.* 2·6

17.

1. The height of a water-fall is 20 ft., and 200 c. ft. of water go over the fall every minute, what will be the H. P. of the water whose modulus is $\frac{2}{3}$? *Ans.* 5.

2. If 80 c. ft. descend upon the wheel per min., what will then be the H. P. ? *Ans.* 2.

3. The breadth of a water-fall is 2 ft., the depth 1 ft., the mean velocity at this part 50 ft. per min., and the height of the fall 16 ft., required the H. P. of the water wheel supposing it to take up all the work in the water? *Ans.* 3·03.

4. If the mean velocity be 40 ft., what will then be the H. P.? *Ans.* 2·4.

5. If the modulus of the wheel in Ex. 3, be ·6, what will then be the H. P.? *Ans.* 1·8.

6. The breadth of a stream is 3 ft., the depth 2 ft., the velocity 40 ft. per min., and the height of the fall 10 ft.; what will be the H. P. of the water-wheel which does $\frac{6}{10}$ of the work applied to it. *Ans.* 2·7.

7. If the height of the fall be 8 ft., what will then be the H. P.? *Ans.* 2·1.

8. The breadth of a stream is 4 ft., the depth 3 ft., the mean velocity 1·5 ft. per sec., and the height of the fall 9 ft.; required the H. P. of the water-wheel whose modulus is ·68. *Ans.* 12·5.

9. If the mean velocity be ·5 ft. per sec., what will then be the H. P.? *Ans.* 4·1.

22 and 23.

1. The area of the piston of a steam engine is 500 sq. in., the mean effective pressure of the steam upon each inch of the piston 30 lbs., the length of the stroke 8 ft., and the number of strokes performed per min. 20; required the H. P. *Ans.* 72·7

2. If the mean pressure of the steam be 20 lbs., what will then be the H. P.? *Ans.* 48·4.

3. What must be the mean effective pressure of the steam, in Ex. 1, so that the H. P. may be 24? *Ans.* 10 *lbs.*

4. The area of the piston of a steam engine is 2800 sq. in.,

the effective pressure upon each inch of the piston 9·lbs., the length of the stroke 7·9 ft., and the number of strokes per min. 11·5; required the H. P. *Ans.* 69·3.

5. If the area of the piston be 700 sq. in., the number of strokes per min. 23, what will then be the H. P.? *Ans.* 34·6

6. The area of the piston of a steam engine is 5000 sq. in., the length of the stroke 10 ft., what must be the effective pressure of the steam per in., so as to give 8 strokes per minute to the piston, or plunger, of a pump, whose section is 9 sq. ft., and stroke 10 ft., the height to which the water is raised being 108 ft.? *Ans.* 12·15 *lbs.*

7. Required the useful H. P. in the last example. *Ans.* 147.

8. The H. P. of an engine = 20, area of the piston = 300 sq. in., length of the stroke = 10 ft. number of strokes per min. = 12; what must be the mean pressure of the steam, allowing that 2 lbs. per sq. in. are necessary to overcome the resistance of friction? *Ans.* 20½ *lbs.*

9. If the H. P. be 15, and the length of the stroke 5 ft., what will then be the pressure? *Ans.* 29·5 *lbs.*

10. The area of the piston of a high pressure engine is 400 in., length of the stroke 3 ft., pressure of the steam 45 lbs., number of strokes per min. 18, resistance of friction and the atmosphere on each inch of the piston 18 lbs.; required the useful H. P., and the water evaporated per min.

Ans. H. P. = 17·67; water evaporated = ·246 c. ft. nearly.

11. Required the same as in the last example, when the pressure of the steam is 48 lbs.

Ans. H. P. = 19·6; water evaporated = ·26 c. ft.

12. If 1 bushel of coals evaporates 11·5 c. ft. of water, what is the duty of the engine in Ex. 10? *Ans.* 27 *millions.*

13. Required the duty of the engine in Ex. 11. *Ans.* 28 *mil.*

28.

1. In a lever of the first kind, the long arm is 2 ft., the short arm 3 in., what must be the power to balance a weight of 64 lbs. *Ans.* 8 *lbs.*

2. What must be the power when the weight is 16 lbs. *Ans.* 2 *lbs.*

3. What will be the weight in Ex. 1, when the power is 14 lbs.? *Ans.* 1 *cwt.*

4. How far from the fulcrum must a power of 7 lbs. be applied so as to balance a weight of 126 lbs. placed at the distance of 2 in. from the fulcrum? *Ans.* 3 *ft.*

5. If the power be 6 lbs., what will then be the distance? *Ans.* 42 *in.*

6. In the handle of a common pump, the piston rod is placed at 3 in. from the fulcrum, a power of 60 lbs. is applied at the distance of 30 in. from the fulcrum; what will be the force lifting the piston? *Ans.* 600 *lbs.*

7. If the power be 47 lbs., what will then be the force? *Ans.* 470 *lbs.*

8. In the great beam of a steam engine, the arm by which the steam acts is 10 ft, and that to which the pump rod is attached 9 ft.; if the pressure of the steam upon the piston be 2 tons, what pressure will be produced upon the plunger of the pump? *Ans.* $2\frac{2}{9}$ *tons.*

9. If the arm of the pump rod be 8 ft., what will then be the pressure on the plunger? *Ans.* $2\frac{1}{2}$ *tons.*

10. What must be the length of the lever of a safety valve (see fig. page 58,) when W = 20 lbs., wt. valve = 6 lbs., V F = 2 in., section of the valve = 4 in., pressure of steam = 50 lbs. the weight of the lever being neglected? *Ans.* 19·4 *in.*

11. If the pressure of the steam is only required at 30 lbs., what must then be the length of the lever? *Ans.* 11·4 *in.*

38.

1. In a wheel and axle, the radius of the wheel is 20 in., and that of the axle 4 in.; if the power applied be 70 lbs., what must be the weight? *Ans.* 350 *lbs.*

2. If the radius of the axle be 3 in., what will then be the weight? *Ans.* $466\frac{2}{3}$ *lbs.*

3. What power must be applied, in Ex. 1, in order to support a weight of 400 lbs.? *Ans.* 80 *lbs.*

4. The handle of a windlass is 2 ft., what must be the radius of the axle, so that a man exerting a pressure of 60 lbs. upon the handle, may raise a tub of coals weighing 4 cwts.? *Ans* 3·2 *in.*

5. If the length of the handle be 20 in., what must then be the radius of the axle? *Ans.* 2·6 *in.*

39.

1. The diameter of the large axle, of a compound wheel and axle, is $\frac{1}{2}$ ft., that of the small one $\frac{1}{8}$ ft., the length of the handle 2 ft., and the weight raised 800 lbs. Required the power. *Ans.* 37·5 *lbs.*

2. If the diameter of the small axle be $\frac{3}{8}$ ft., what will then be the power? *Ans.* 12·5 *lbs.*

3. If the diameter of the large axle, in Ex. 1, be $\frac{1}{4}$ ft., what will then be the power? *Ans.* 12·5 *lbs.*

4. If the power, in Ex. 1, be 75 lbs., what will be the weight? *Ans.* 1600 *lbs.*

41.

1. If there are 3 moveable pulleys (see fig. page 69), required P, when W = 6 lbs. *Ans.* 2 *lbs.*

2. Required P, when there are 6 moveable pulleys. *Ans.* 1 *lb.*

3. Let there be 1 moveable pulley, weighing 4 lbs.; required P when W = 180 lbs. *Ans.* 92 *lbs.*

4. When there are 2 moveable pulleys, what will then be the power? *Ans.* 48 *lbs.*

42.

1. The length of an inclined plane is 80 ft., height 2 ft., the weight of the body 2 tons; what pressure will be required to sustain the body on the plane, friction being neglected? *Ans.* 1 *cwt.*

2. If the height of the plane be 3 ft.; what will then be the pressure? *Ans.* $1\frac{1}{2}$ *cwts.*

3. If the friction in Ex. 1, be $\frac{1}{30}$ of the load; what must be the pressure to move the body up the plane? *Ans.* $261\frac{1}{3}$ *lbs.*

4. If the friction be $\frac{1}{20}$ of the load, what will then be the pressure? *Ans.* 3 *cwts.*

44 and 45.

1. The lever of a simple screw is 3 ft. long, the thickness of the threads $\frac{1}{2}$ in., and the pressure applied to the lever 60 lbs.; required the pressure on the press board, friction being neglected. *Ans.* 12·1 *tons.*

2. When the lever is 2 ft., and the thickness of the threads $\frac{3}{4}$ in., what will then be the pressure? *Ans.* 5·3 *tons.*

3. If the thickness of the threads in Ex. 1, be $\frac{2}{3}$ in. what will then be the pressure? *Ans.* 9 *tons.*

4. In a compound screw, the length of the lever is 3 ft., the distance between the threads of the large screw $\frac{1}{2}$ in., and that of the small one $\frac{3}{7}$ in.; if 60 lbs. be applied to the extremity of the lever, what will be the pressure produced on the press board? *Ans.* 84·7 *tons.*

5. If the distance between the threads of the small screw be $\frac{3}{8}$ in., and the power applied 20 lbs., what will then be the pressure? *Ans.* 16·1 *tons.*

6. What is the advantage of pressure, in Ex. 4? *Ans.* 3165.

47.

1. The area of the small piston of a hydrostatic press is 1 sq. in., that of the large one 200 sq. in., the lever is 30 in. long, and the piston rod is placed at 2 in. from the fulcrum; if a pressure of 56 lbs. be applied to the lever, what pressure will be produced upon the large piston? *Ans.* 75 *tons.*

2. If the area of the large piston be 40 sq. in., what will then be the pressure? *Ans.* 15 *tons.*

3. If the surfaces of the pistons in Ex. 1, be $\frac{2}{3}$ and 400 sq. in. respectively, what will then be the pressure? *Ans.* 225 *tons.*

50 and 52,

1. What velocity will a falling body have at the end of 5 seconds? *Ans.* $160\frac{5}{6}$ *ft.*

2. What velocity will a body acquire in 2 sec.? *Ans.* $64\frac{1}{3}$ *ft.*

3. In what time will a falling body acquire a velocity of 193 ft.? *Ans.* 6 *sec.*

4. In what time will a body acquire a velocity of 386 ft.? *Ans.* 12 *sec.*

5. A stone was 2 seconds in falling from the top of a tower; what is its height? *Ans.* 64⅓ *ft.*

6. From what height will a body fall in 5 sec.? *Ans.* 402 *ft.*

7. If a body be projected upwards with a velocity of 96½ ft.; to what height will it go? *Ans.* 144¾ *ft.*

8. To what height will it ascend in 2 sec.? *Ans.* 128⅔ *ft.*

9. A body has a velocity of 193 ft., from what height must it fall to acquire this velocity? *Ans.* 579 *ft.*

10. If the weight of the body in the last example be 8 lbs.; required the work accumulated in it. *Ans.* 4632.

11. A body weighing 4 lbs. has a velocity of 386 ft. per sec.; how many units of work had been done upon it? *Ans.* 9264.

70 and 71.

1. Let O H = 6 ft., O A = 3 ft., (see fig. page 101,) and the weight of the gate = 4 cwts.; required the pressure upon the pin O. *Ans.* 5·6 *cwts.*

2. If the weight of the gate be 3 cwts., what will then be the pressure? *Ans.* 4·2 *cwts.*

3. If the height O A of the gate, in Ex. 1, be 5ft., what will then be the pressure? *Ans.* 4·6 *cwts.*

4. Let O R = 5 ft., (see fig. page 102,) O V = 2 ft., R P = 3 ft., length of the pier = 1 ft., weight of a c. ft. of the material = 120 lbs., P = 3000 lbs., and its direction 45° inclined to the horizon; will the pier stand or fall? *Ans.* It will stand.

5. Required P so that the pier may be upon the point of overturning. *Ans.* 3400 *lbs.*

6. Will the pier, in Ex. 4, stand when the direction of P is 30°? *Ans.* It will fall.

76 and 78.

1. Required the pressure on a flood-gate, whose breadth is 10 ft., and depth 6 ft. *Ans.* 11250 *lbs.*

2. If the depth be 9 ft., what then will be the pressure? *Ans.* 25312 *lbs.*

3. How far from the bottom will the whole pressure in Ex. 1 act? *Ans.* 2*ft.*

4. The height of a rectangular embankment is 9 ft., the thickness 4 ft., and the weight of a c. ft. of the material 120 lbs.; will the embankment stand or fall when the water is at the brim? *Ans.* It will stand.

5. Will it stand or fall when the thickness is 3 ft. *Ans.* It will fall.

82.

1. What load will just sink a barge which is 10 ft. long, 4 ft. broad, and 3 ft. deep, outside measure, and the thickness of the planking 2 inches, and the weight of a c. ft. of timber 50 lbs. *Ans.* 2·9 *tons.*

2. How far would the barge, in the last example, sink with a load of 2 tons? *Ans.* 2·18*ft.*

MISCELLANEOUS EXAMPLES.

1. What will be the H. P. of an engine which performs 17894000 units of work per hour? *Ans.* 9·03.

2. How many cubic feet of water will an engine of 7 H. P. pump per hour, to the height of 100 ft., supposing $\frac{1}{3}$ of the work to be destroyed by friction, &c.? *Ans.* 1478·4

3. How many strokes per min., will an engine of 80 effective H. P., give to the piston of a pump whose area is 2 sq. ft., length of each stroke 5 ft., and the height to which the water is raised 60 fathoms? *Ans.* 11·7

4. The shaft of a pit is to be sunk 40 fathoms, its section contains 30 sq. ft., how many units of work will it take to raise the material, supposing each c. ft. to weigh 100 lbs.? *Ans.* 86400000.

5. If the depth be 5 fathoms, what will then be the work? *Ans.* 1350000.

6. A wall is 40 ft. long, 20 ft. high, and 3 ft. thick; required the work in raising the material, supposing each c. ft. to weigh 120 lbs. *Ans*. 2880000.

7. A cistern is 12 ft. long, 6 ft. broad, and 10 ft. deep, how much work will it take to fill the cistern with water, when the bottom is 30 ft. from the level of the water in the well? *Ans*. 1575000

8. If the depth of the cistern be 12 ft., what will then be the work? *Ans*. 1944000.

9. If the cistern, in Ex. 7, is to be filled 3 times every hour, how many H. P. will it take to do it? *Ans*. 2·38.

10. A horse turns a whim gin, and exerts a traction of 160 lbs.; how many revolutions will he make per min., when he moves in a circle whose diameter is 20 ft., allowing that he does 22000 units of work per minute? *Ans*. 2·17.

11. The weights of two bodies P and W are 4 and 8 lbs. respectively, and the distance apart 15 in.; required the distance of the centre of gravity from W. *Ans*. 5 *in*.

12. Required the same as in the last example, when the weight of P is 2 lbs. *Ans*. 3 *in*.

FINIS.

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